8.4 REVIEW WORKSHEET

Pre-Calculus

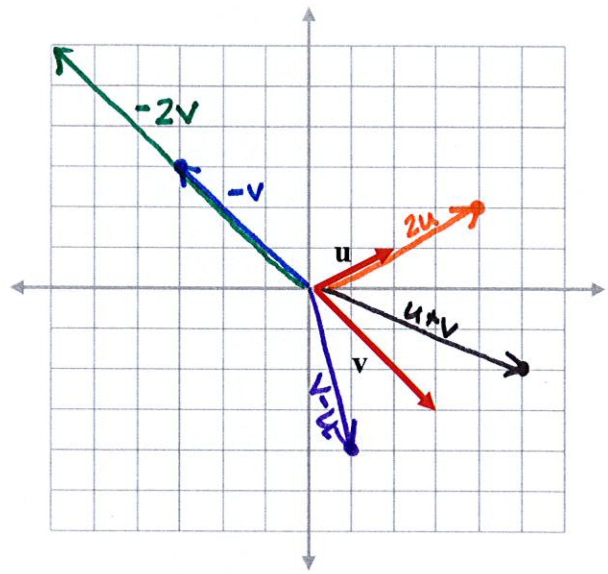
Name:__________________________________

Sketch the vector indicated.

$u = \langle 2, 1 \rangle$

$v = \langle 3, -3 \rangle$

1. $2u$ $\langle 4, 2 \rangle$
2. $-v$ $\langle -3, 3 \rangle$
3. $-2v$ $\langle -6, 6 \rangle$
4. $v - u$ $\langle 1, -4 \rangle$
5. $u + v$ $\langle 5, -2 \rangle$



Express the vector with initial point P and terminal point Q in component form.

6. $P(3,\ 2),\ Q(10,\ 6)$

$\langle 3-10, 2-6 \rangle = \langle -7, -4 \rangle$

7. $P(-2,\ 4),\ Q(-8,\ -3)$

$\langle -2-(-8),\ 4-(-3) \rangle = \langle 6, 7 \rangle$

Find $-2\boldsymbol{u}$, $\boldsymbol{u} + \boldsymbol{v}$, $2\boldsymbol{v} - \boldsymbol{u}$, $|\mathbf{u}|$, $|\mathbf{v}|$, and $|\mathbf{u} - \mathbf{v}|$ for the given vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.

8. $\mathbf{u} = \langle -1,\ 4 \rangle,\ \mathbf{v} = \langle 3, -7 \rangle$

$-2u = \langle 2, -8 \rangle$

$u + v = \langle 2, -3 \rangle$

$2v - u = \langle 6, -14 \rangle - \langle -1, 4 \rangle = \langle 7, -18 \rangle$

$|u| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

$|v| = \sqrt{(3)^2 + (-7)^2} = \sqrt{58}$

$|u - v|$ 1.) $u - v = \langle -4, 11 \rangle$

2.) now find length

$\sqrt{(-4)^2 + 11^2} = \sqrt{16 + 121}$

$= \sqrt{137}$

9. $\mathbf{u} = \mathbf{i} + \mathbf{j},\ \mathbf{v} = \mathbf{i} - \mathbf{j}$

$-2u = -2i - 2j$

$u + v = 2i$

$2v - u = 2i - 2j - (i + j) = i - 3j$

$|u| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$|v| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$|u - v|$ 1.) $u - v = 2j$ or $0i + 2j$

2.) now find length

$\sqrt{0^2 + 2^2} = 2$

Find $-2\boldsymbol{u}$, $\boldsymbol{u} + \boldsymbol{v}$, $2\boldsymbol{v} - \boldsymbol{u}$, $|\mathbf{u}|$, $|\mathbf{v}|$, and $|\mathbf{u} - \mathbf{v}|$ for the given vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.

10. $\mathbf{u} = -\mathbf{i} + 2\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

$-2u = 2i - 4j$

$u + v = i - j$

$2v - u = 4i - 6j - (-i + 2j) = 5i - 8j$

$|u| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$

$|v| = \sqrt{(2)^2 + (-3)^2} = \sqrt{13}$

$|u - v| =$ 1.) $u - v = -3i + 5j$

2.) now find length

$\sqrt{(-3)^2 + 5^2} = \sqrt{34}$

11. $\mathbf{u} = \langle -7, 2 \rangle$, $\mathbf{v} = \langle -3, -1 \rangle$

$-2u = \langle 14, -4 \rangle$

$u + v = \langle -10, 1 \rangle$

$2v - u = \langle -6, -2 \rangle - \langle -7, 2 \rangle = \langle 1, -4 \rangle$

$|u| = \sqrt{(-7)^2 + 2^2} = \sqrt{53}$

$|v| = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$

$|u - v| =$ 1.) $u - v = \langle -4, 3 \rangle$

2.) now find length

$\sqrt{(-4)^2 + (3)^2} = \sqrt{25} = 5$

Find the horizontal and vertical components of the vector with the given length and direction. Write your answer in component form AND in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

12. $|\mathbf{v}| = 5$, $\theta = \frac{2\pi}{3}$

$v_1 = |v|\cos\theta = 5\cos\frac{2\pi}{3} = 5\left(-\frac{1}{2}\right) = -\frac{5}{2}$

$v_2 = |v|\sin\theta = 5\sin\frac{2\pi}{3} = 5\left(\frac{\sqrt{3}}{2}\right) = \frac{5\sqrt{3}}{2}$

$\left\langle -\frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle$ $\quad -\frac{5}{2}i + \frac{5\sqrt{3}}{2}j$

13. $|\mathbf{v}| = \sqrt{3}$, $\theta = 240°$

$v_1 = |v|\cos\theta = \sqrt{3}\cos 240° = \sqrt{3}\left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{2}$

$v_2 = |v|\sin\theta = \sqrt{3}\sin 240° = \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{3}{2}$

$\left\langle -\frac{\sqrt{3}}{2}, -\frac{3}{2} \right\rangle$ $\quad -\frac{\sqrt{3}}{2}i - \frac{3}{2}j$

14. $|\mathbf{v}| = 6$, $\theta = 225°$

$v_1 = |v|\cos\theta = 6\cos 225° = 6\left(-\frac{\sqrt{2}}{2}\right) = -3\sqrt{2}$

$v_2 = |v|\sin\theta = 6\sin 225° = 6\left(-\frac{\sqrt{2}}{2}\right) = -3\sqrt{2}$

$\langle -3\sqrt{2}, -3\sqrt{2} \rangle$ $\quad -3\sqrt{2}i - 3\sqrt{2}j$

15. $|\mathbf{v}| = 4$, $\theta = \frac{\pi}{2}$

$v_1 = |v|\cos\theta = 4\cos\frac{\pi}{2} = 4(0) = 0$

$v_2 = |v|\sin\theta = 4\sin\frac{\pi}{2} = 4(1) = 4$

$\langle 0, 4 \rangle$ $\quad 0i + 4j$ or simply $4j$

Find the magnitude and direction of the vector. Write your answer in radians.

16. $\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$

mag: $\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$

direction: $\tan^{-1}\left(\frac{-\sqrt{2}/2}{\sqrt{2}/2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

$Q4 \to \frac{7\pi}{4}$

17. $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j}$

mag: $\sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

direction: $\tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$ $\quad Q1 \to \frac{\pi}{4}$

18. $\mathbf{u} = -\mathbf{i} - \sqrt{3}\mathbf{j}$

mag: $\sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$

direction: $\tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$Q3 \to \frac{4\pi}{3}$

19. $\langle -3\sqrt{3}, 3 \rangle$

mag: $\sqrt{(-3\sqrt{3})^2 + 3^2} = \sqrt{27 + 9} = \sqrt{36} = 6$

direction: $\tan^{-1}\left(\frac{3}{-3\sqrt{3}}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ $\quad Q2 \to \frac{5\pi}{6}$