Math 12H Finding limits algebraically

p. 897 4-26 even, 33

4) $\lim_{x\to 3} (x^3+2)(x^2-5x) = 29 \cdot (-6) = \boxed{-174}$

6) $\lim_{x\to 1} \left(\frac{x^4+x^2-6}{x^4+2x+3}\right)^2 = \left(\frac{1+1-6}{1+2+3}\right)^2 = \left(\frac{-4}{6}\right)^2 = \left(\frac{-2}{3}\right)^2 = \boxed{\frac{4}{9}}$

8) $\lim_{u\to -2} \sqrt{u^4+3(u)+6} = \sqrt{16-6+6} = \boxed{4}$

10) $\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4} = \lim_{x\to -4} \frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x-1)} = \lim_{x\to -4} \frac{(-4+1)}{(-4-1)} = \frac{-3}{-5} = \boxed{\frac{3}{5}}$

can't plug in

12) $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} = \frac{3}{2} = \boxed{1.5}$

14) $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \frac{1+h-1}{h(\sqrt{1+h}+1)} = \frac{h}{h(\sqrt{1+h}+1)}$

$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$

16) $\lim_{x\to 2} \frac{x^4-16}{x-2} = \lim_{x\to 2} \frac{(x^2+4)(x^2-4)}{x-2} = \lim_{x\to 2} \frac{(x^2+4)(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$= \lim_{x\to 2} (x^2+4)(x+2) = 8(4) = \boxed{32}$

18) $\lim_{h\to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

$= \lim_{h\to 0} \dfrac{3\cdot(3+h)\left[(3+h)^{-1} - 3^{-1}\right]}{3\cdot(3+h)\cdot h}$

$= \lim_{h\to 0} \dfrac{3(3+h)(3+h)^{-1} - 3^{-1}(3)(3+h)}{3h(3+h)}$

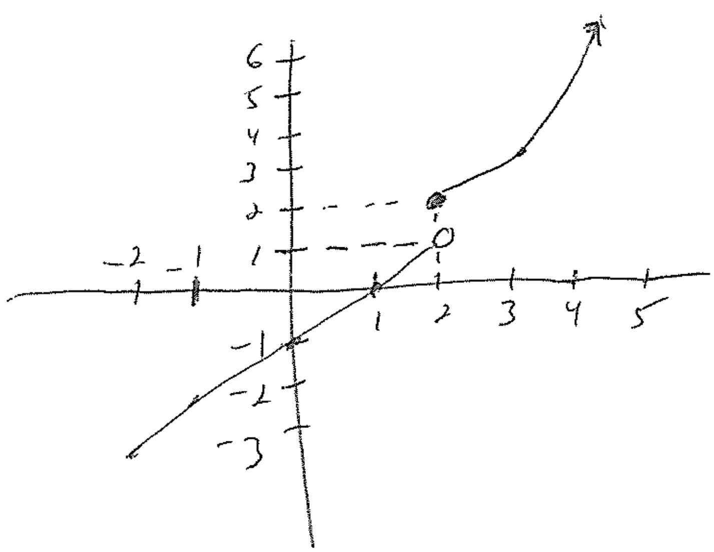
$= \lim_{h\to 0} \dfrac{3-(3+h)}{3h(3+h)} = \lim_{h\to 0} \dfrac{3-3-h}{3h(3+h)} = \lim_{h\to 0} \dfrac{-h}{3h(3+h)}$

$= \lim_{h\to 0} \dfrac{-1}{3(3+h)} = \boxed{-\dfrac{1}{9}.}$

20) $\lim_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right) = \lim_{t\to 0} \left(\dfrac{1}{T} - \dfrac{1}{T(T+1)}\right) = \lim_{t\to 0} \left(\dfrac{1\,(T+1)}{T(T+1)} - \dfrac{1}{T(T+1)}\right)$

$= \lim_{T\to 0} \dfrac{T+1-1}{T(T+1)} = \lim_{T\to 0} \dfrac{T}{T(T+1)} = \lim_{T\to 0} \dfrac{1}{T+1} = \dfrac{1}{0+1} = \boxed{1}$

33) $f(x) = \begin{cases} x-1 & \text{if } x < 2 \\ x^2-4x+6 & \text{if } x \geq 2 \end{cases}$




a) $\lim_{x\to 2^-} f(x) = 1$

$\lim_{x\to 2^+} f(x) = 2$

b) $\lim_{x\to 2} f(x) =$ D.N.E.

Left limit $1 \neq$ Right limit $2$