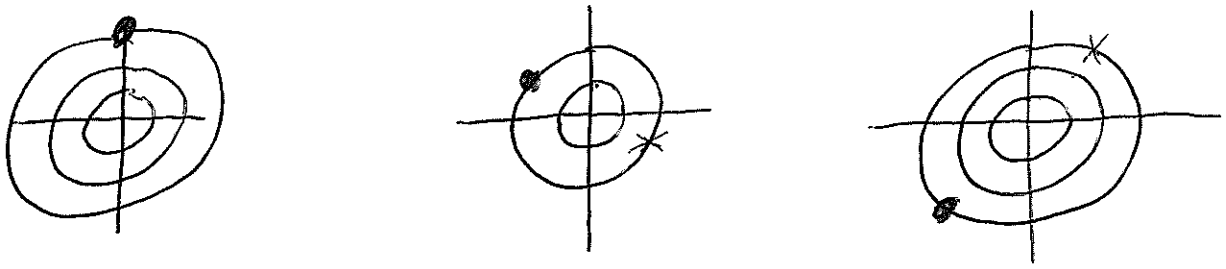
Math 12 H Answer Key – Review of Parametric / Polar Equations

1) Plot $(3, \frac{\pi}{2})$ $(-2, \frac{11\pi}{6})$ $(-3, -\frac{7\pi}{4})$



Remember $-r$ reflects the angle 180°,

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2) Give 2 sets of coordinates for each with $r>0$ and $r<0$

a) $(8, \frac{5\pi}{6}) = (8, \frac{17\pi}{6}) = (8, -\frac{7\pi}{6}) = (-8, \frac{11\pi}{6}) = (-8, \frac{23\pi}{6})$

$+r = \pm 2\pi = \pm \frac{12\pi}{6}$ $\quad$ $-r = \pm 1\pi$ and then $\pm 2\pi$

b) $(-2, \pi) = (-2, 3\pi) = (-2, 5\pi) = (2, 2\pi) = (2, 4\pi)$

c) $(-1, -\frac{3\pi}{4}) = (-1, \frac{5\pi}{4}) = (-1, \frac{13\pi}{4}) = (1, \frac{\pi}{4}) = (1, \frac{9\pi}{4})$

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3) Find the rectangular coordinates for each:

a) $(8, \frac{7\pi}{6}) = 8(\underset{210^\circ}{\cos}\frac{7\pi}{6}, \underset{210^\circ}{\sin}\frac{7\pi}{6}) = 8(-\frac{\sqrt{3}}{2}, -\frac{1}{2}) = \boxed{(-4\sqrt{3}, -4)}$

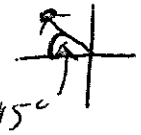
b) $(-6, \frac{3\pi}{2}) = -6(\underset{(0)}{\cos}270, \underset{(-1)}{\sin}270) = \boxed{(0, 6)}$

c) $(-1, -\frac{5\pi}{3}) = -1(\cos(-300), \overset{-\sin(300)}{\sin(-300)}) = \boxed{(-\frac{1}{2}, -\frac{\sqrt{3}}{2})}$

$(\frac{1}{2})$ $\quad$ $, -(-\frac{\sqrt{3}}{2})$

#4) Convert to Polar Coordinates Find r and θ

a) $(-4,4)$ $r = \sqrt{(-4)^2+4^2} = \sqrt{32} = \sqrt{16}\cdot\sqrt{2} = 4\sqrt{2}$



$\tan^{-1}\left(\frac{4}{4}\right) = 45° = \theta_r$ $\theta = 135°$ $(-4,4) = (4\sqrt{2}, 135°) = \left(4\sqrt{2}, \frac{3\pi}{4}\right)$

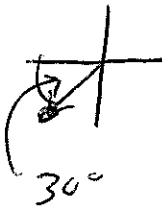
4b) $(0,-8)$ $r = \sqrt{0^2+(-8)^2} = \sqrt{64} = 8$

$\theta = 270°$ $(0,-8) = (8, 270°) = \left(8, \frac{3\pi}{2}\right)$

1c) $(-1,-\sqrt{3})$ $r = \sqrt{(-1)^2+(-\sqrt{3})^2} = \sqrt{4} = 2$ $(-1,-\sqrt{3}) = (2, 240°) = \left(2, \frac{4\pi}{3}\right)$

$\tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = 60° = \theta_r$ $\theta = 240°$

1d) $(-\sqrt{6},-\sqrt{2})$ $r = \sqrt{(-\sqrt{6})^2+(-\sqrt{2})^2} = \sqrt{8} = 2\sqrt{2}$



$\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{6}}\right) = 30° = \theta_r$ $(-\sqrt{6},-\sqrt{2}) = (2\sqrt{2}, 210°) = \left(2\sqrt{2}, \frac{7\pi}{6}\right)$

$\theta = 210° = \frac{7\pi}{6}$

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5) Change to rectangular Equations

$r = 5$ $(r)^2 = (5)^2$ $r^2 = 25$ $x^2+y^2=25$

5b) $\theta = \frac{5\pi}{4}$ $\tan\theta = \tan\frac{5\pi}{4}$

$\frac{r\sin\theta}{r\cos\theta} = 1$ $\frac{y}{x} = \frac{1}{1}$ → $y = x$

5c) $r = 5\cos\theta$ $\quad r(r = 5\cos\theta)$

$r^2 = 5r\cos\theta \rightarrow x^2 + y^2 = 5x$

$x^2 - 5x + y^2 = 0 \rightarrow \boxed{(x-2.5)^2 + y^2 = 6.25}$

5d) $r\sin^2\theta = 3\cos\theta$ multiply both sides by r.

$r^2\sin^2\theta = 3r\cos\theta$

$\boxed{y^2 = 3x}$

5e) $r = 1 + \sin\theta \leftarrow$ multiply by r

$r^2 = r + r\sin\theta \rightarrow x^2 + y^2 = r + y \rightarrow$

$(x^2 + y^2 - y) = r \leftarrow$ Square both sides

$(x^2 + y^2 - y)^2 = r^2 \rightarrow \boxed{(x^2 + y^2 - y)^2 = x^2 + y^2}$

5f) $r = \dfrac{2}{1+\cos\theta}$ $\quad r + r\cos\theta = 2$

$r + x = 2$

$r = 2 - x \qquad (r)^2 = (2-x)^2$

$x^2 + y^2 = (4 - 4x + x^2) \rightarrow \boxed{y^2 = -4x + 4}$

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6) Convert to a polar equation.

a) $x^2 + y^2 = 49 \rightarrow r^2 = 49 \rightarrow \boxed{r = 7}$

6b) $y = -4 \rightarrow \frac{r\sin\theta}{\sin\theta} = \frac{-4}{\sin\theta}$ $r = \frac{-4}{\sin\theta}$ $\boxed{r = -4\csc\theta}$

6c) $x^3 = 4y^2$ $x \cdot x^2 = 4y^2$ $x = \frac{4y^2}{x^2}$

$r\cos\theta = \frac{4r^2\sin^2\theta}{r^2\cos^2\theta}$ $r\cos\theta = 4\tan^2\theta$

$\frac{r\cos\theta}{\cos\theta} = \frac{4\tan^2\theta}{\cos\theta}$ $\boxed{r = 4\tan^2\theta\sec\theta}$

6d) $y = x$ $\frac{y}{x} = \frac{x}{x}$ $\frac{y}{x} = 1 \rightarrow \frac{r\sin\theta}{r\cos\theta} = 1 \rightarrow$

$\tan\theta = 1$ $\tan^{-1}(\tan\theta) = \tan^{-1}(1) \rightarrow \boxed{\theta = \frac{\pi}{4}}$

6e) $x^2 + y^2 - 4x + 6y = 0$

$r^2 - 4r\cos\theta + 6r\sin\theta = 0$

$\frac{r}{r}(r - 4\cos\theta + 6\sin\theta) = \frac{0}{r}$

$r - 4\cos\theta + 6\sin\theta = 0 \rightarrow \boxed{r = 4\cos\theta - 6\sin\theta}$

6f) $x^2 - y^2 = 1$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2 \cdot \cos 2\theta = 1 \rightarrow r^2 = \frac{1}{\cos 2\theta}$

$\rightarrow r^2 = \sec 2\theta$ $\boxed{r = \pm\sqrt{\sec 2\theta}}$

7) Convert to a rectangular equation

a) $x = 4t^2 \quad y = 8t^3$

$\frac{x}{4} = t^2 \quad t = \pm\sqrt{\frac{x}{4}} = \pm\frac{\sqrt{x}}{2}$

$y = 8\left(\pm\frac{\sqrt{x}}{2}\right)^3$

$= \pm\frac{8 \cdot x\sqrt{x}}{8} = \boxed{y = \pm x\sqrt{x}}$

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b) $x = 2\cos t \quad y = 3\sin t \quad 0 \le t^2 \le 2\pi$

$\cos t = \frac{x}{2} \quad \sin t = \frac{y}{3}$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = \cos^2 t + \sin^2 t = 1$

$= \boxed{\frac{1}{4}x^2 + \frac{1}{9}y^2 = 1}$

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c) $x = \cos t \quad y = \cos 2t$

$x^2 = \cos^2 t \rightarrow$ then $2x^2 - 1 = \underbrace{2\cos^2 t - 1}_{\text{identity}} = \cos 2t = y$

$= \boxed{2x^2 - 1 = y}$ where $-1 \le x \le 1$

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d) $x = \sin^2 t \quad y = \sin^4 t$

$x^2 = \sin^4 t = y \rightarrow \boxed{y = x^2}$ since $x = \sin^2 t$

$0 \le 1 \le x$

6e) $x = \frac{3}{\sqrt{3-t}}$ $\quad y = \frac{t-3}{t} \rightarrow yt = t-3$

$yt - t = -3$

$t(y-1) = -3 \rightarrow t = \frac{-3}{y-1}$

$$x = \frac{3}{\sqrt{3-\left(\frac{-3}{y-1}\right)}} = \frac{3}{\sqrt{\frac{3}{1}+\frac{3}{y-1}}} = \frac{3}{\sqrt{\frac{3y-3+3}{y-1}}} = \frac{3}{\sqrt{\frac{3y}{y-1}}}$$

$$x^2 = \frac{9}{1} \div \frac{3y}{y-1} \rightarrow x^2 = \frac{9}{1} \cdot \frac{y-1}{3y} \rightarrow \frac{x^2}{1} = \frac{3y-3}{y}$$

$\rightarrow x^2y = 3y-3 \rightarrow x^2y - 3y = -3 \rightarrow y(x^2-3) = -3$

$\rightarrow \boxed{y = \frac{-3}{x^2-3}}$

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8) Find the parametric equation for a line with the given properties. Convert to a rectangular equation.

a) slope $= \frac{1}{2}$ point $= (4,-1)$

$\boxed{x = 4+2t \quad y = -1+1t}$ parametric

$t = y+1$ $\quad x = 4+2(y+1)$ $\quad y = \frac{x-6}{2}$

$x = 4+2y+2$

$x = 6+2y$

$\boxed{y = \frac{1}{2}x - 3}$ Rectangular

8b) Passes through $(6,7)$ $(7,8)$ $\quad m = \frac{8-7}{7-6} = \frac{1}{1}$

$\boxed{x = 6+t \quad y = 7+t}$

$x-6 = t$ $\quad y = 7+(x-6)$ $\quad y = 7+x-6$

$\boxed{y = x+1}$ Rectangular

9) What is the shape of the polar equation

$r = 2 + 2\sin\theta$

form $= a + b\sin\theta$ = Limacon where $a = b$

= **a cardioid**

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10) What is the maximum value for r in the equation $r = 6(4 - 5\cdot\underbrace{\sin\theta}_{(-1)})$

$= 6(4 - (5)(-1))$

$= 6(9) = \boxed{54}$

Sin θ values

min = −1, max = 1

can also graph for solution.