Matrices Review 12 H

1) $2D - A = 2\begin{bmatrix} 4 & 7 \\ -1 & 3 \\ -2 & -5 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 0 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 14 \\ -2 & 6 \\ -4 & -10 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 0 \\ 4 & 4 \end{bmatrix}$

$= \begin{bmatrix} 9 & 13 \\ -4 & 6 \\ -8 & -14 \end{bmatrix}$

2) $A \cdot C$ $\begin{bmatrix} -1 & 1 \\ 2 & 0 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \\ 5 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -3+5 & 1+4 & -2+1 \\ 2 & 5 & -1 \\ 6+0 & -2+0 & 4+0 \\ 6 & -2 & 4 \\ 32 & 12 & 12 \\ +12+20 & -4+16 & 8+4 \end{bmatrix}$

Row Elements of A × Column C

$3\times 2 \cdot 2\times 3$ ✓ new size

3) $B^{-1} = B$ inverse $= \frac{1}{|B|}\cdot\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$B = \begin{bmatrix} 2 & 5 \\ -5 & -13 \end{bmatrix}$ $|B| = 2(-13) - (5)(-5)$ $= -26 - -25$ $= -1$

$B^{-1} = \frac{1}{-1}\begin{bmatrix} -13 & -5 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 13 & 5 \\ -5 & -2 \end{bmatrix}$

4) $\frac{X}{2} + 4A = 3D$

$\frac{X}{2} = 3D - 4A$

$X = 2(3D - 4A)$

$3D - 4A$

$\begin{bmatrix} 12 & 21 \\ -3 & 9 \\ -6 & -15 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ 8 & 0 \\ 16 & 16 \end{bmatrix} = \begin{bmatrix} 16 & 17 \\ -11 & 9 \\ -22 & -31 \end{bmatrix}$

$X = 2(3D - 4A) = \begin{bmatrix} 32 & 34 \\ -22 & 18 \\ \cdots & \cdots \end{bmatrix} \Leftrightarrow X$

5) $|E| = \begin{vmatrix} 0 & -2 & 2 & 0 & -2 \\ 3 & 1 & 3 & 3 & 1 \\ 1 & -2 & 3 & 1 & -2 \end{vmatrix}$

$= (0 + -6 + -12) - (-18 + 0 + 2) = (-18) - (-16) = -2$

6) $E^{-1} =$

$\xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 0 & -2 & 2 & 1 & 0 & 0 \\ 3 & 1 & 3 & 0 & 1 & 0 \\ 1 & -2 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-3R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & -4 & 5 & 1 & 0 & 1 \\ 0 & 7 & -6 & 0 & 1 & -3 \\ 1 & -2 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-R_1 + R_3}$

$\xrightarrow{\substack{+2R_2 \\ -7R_3}} \left[\begin{array}{ccc|ccc} 1 & -4 & 5 & 1 & 0 & 1 \\ 0 & 7 & -6 & 0 & 1 & -3 \\ 0 & 2 & -2 & -1 & 0 & 0 \end{array}\right] \xrightarrow{R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & -4 & 5 & 1 & 0 & 1 \\ 0 & 14 & -12 & 0 & 2 & -6 \\ 0 & -14 & 14 & 7 & 0 & 0 \end{array}\right] \xrightarrow{\substack{R_2/2 \\ R_3/2}} \left[\begin{array}{ccc|ccc} 1 & -4 & 5 & 1 & 0 & 1 \\ 0 & 14 & -12 & 0 & 2 & -6 \\ 0 & 0 & 2 & 7 & 2 & -6 \end{array}\right]$

$\xrightarrow{\substack{-5R_3 + R_1 \\ +R_2 \\ ; R_3 + R_2}} \left[\begin{array}{ccc|ccc} 1 & -4 & 5 & 1 & 0 & 1 \\ 0 & 7 & -6 & 0 & 1 & -3 \\ 0 & 0 & 1 & \frac{7}{2} & 1 & -3 \end{array}\right] \xrightarrow{R_2/7} \left[\begin{array}{ccc|ccc} 1 & -4 & 0 & -\frac{33}{2} & -5 & 16 \\ 0 & 7 & 0 & 21 & 7 & -21 \\ 0 & 0 & 1 & \frac{7}{2} & 1 & -3 \end{array}\right]$

$\xrightarrow{4R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & -4 & 0 & -\frac{33}{2} & -5 & 16 \\ 0 & 1 & 0 & 3 & 1 & -3 \\ 0 & 0 & 1 & \frac{7}{2} & 1 & -3 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{9}{2} & -1 & 4 \\ 0 & 1 & 0 & 3 & 1 & -3 \\ 0 & 0 & 1 & \frac{7}{2} & 1 & -3 \end{array}\right]$

$E^{-1} = \begin{bmatrix} -\frac{9}{2} & -1 & 4 \\ 3 & 1 & -3 \\ \frac{7}{2} & 1 & -3 \end{bmatrix}$

Check

$E \cdot E^{-1} = E^{-1} \cdot E = I$ ✓

Math 12 H Review (must equal)

7) $B \cdot E = 2\times\underline{3} \cdot \underline{3}\times 3$

# elements in columns of first matrix must equal # elements in row of 2nd matrix

8) Area $= \pm\frac{1}{2}\begin{vmatrix} -1 & 4 & 1 & -1 & 4 \\ 2 & 6 & 1 & 2 & 6 \\ 1 & 0 & 1 & 1 & 0 \end{vmatrix} = -\frac{1}{2}(-16) = \boxed{8}$

$= (-6+4+0) - (8+0+6) = (-2) - (14) = -16$

9) $\begin{vmatrix} 7-2x & -1 \\ 5x+2 & 4 \end{vmatrix} = 3$

$4(7-2x) - [(-1)(5x+2)] = 3$

$28 - 8x - [-5x - 2] = 3$

$28 - 8x + 5x + 2 = 3$

$-3x + 30 = 3$

$-3x = -27$

$\boxed{x = 9}$

10) $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & -6 & -1 \\ 4 & 3 & 5 \end{bmatrix}$

$C_{32}$

$\begin{bmatrix} -1 & 5 & 1 \\ 2 & -6 & -1 \\ 4 & 3 & 5 \end{bmatrix}$

$(-1)(-1) - (1)(2)$

$= 1 - 2$

$= (-1)^{3+2} \cdot \begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} = (-1)^5 \cdot (1-2)$

$= -1 \cdot -1 = \boxed{1}$

Points $= (2,-5)\ (-1,-1)$

11) $\begin{vmatrix} x & y & 1 & x & y \\ 2 & -5 & 1 & 2 & -5 \\ -1 & -1 & 1 & -1 & -1 \end{vmatrix} = 0$

$(-5x - y - 2) - (2y - x + 5) = 0$

$-5x - y - 2 - 2y + x - 5 = 0$

$-3y - 4x - 7 = 0$

$= \boxed{3y + 4x + 7 = 0}$ ← Equation

$\rightarrow 3y = -4x - 7$

$\rightarrow \boxed{y = -\frac{4}{3}x - \frac{7}{3}}$ ← standard form

12) $\begin{vmatrix} 2 & 1 \\ 5 & -3 \end{vmatrix} = -6 - 5 = -11$ $|D| = -11$ $x = \frac{-22}{-11} = 2$

$|D_x| = \begin{vmatrix} 7 & 1 \\ 1 & -3 \end{vmatrix} = -21 - 1 = -22$ $|D_x| = -22$ $y = \frac{-33}{-11} = 3$

$|D_y| = \begin{vmatrix} 2 & 7 \\ 5 & 1 \end{vmatrix} = 2 - 35 = -33$ $|D_y| = -33$

$x = 2,\ y = 3$

15) $-3x + 5y + 2z = -1$

$2y + 4z = 6$

$-z = 2$ back fill

$2y + -8 = 6$

$2y = 14$

$y = 7$

$z = -2$

$-3x + 5(7) + 2(-2) = -1$

$-3x + 35 - 4 = -1$

$-3x + 31 = -1$

$\frac{-3x}{-3} = \frac{-32}{-3}$

$x = 10\frac{2}{3}$

13) $\begin{vmatrix} 3 & 2 & -5 & 3 & 2 \\ 2 & 4 & 1 & 2 & 4 \\ 1 & -6 & -4 & 1 & -6 \end{vmatrix}$

$(-48 + 2 + 60) - (-16 + -18 + -20)$

$= (14) - (-54) = 68 = |D|$

$|D_x| = \begin{vmatrix} -10 & 2 & -5 & -10 & 2 \\ 0 & 4 & 1 & 0 & 4 \\ -3 & -6 & -4 & -3 & -6 \end{vmatrix}$

$= (160 + -6 + 0) - (0 + 60 + 60)$

$= (154) - (120) = 34$

$|D_y| = \begin{vmatrix} 3 & -10 & -5 & 3 & -10 \\ 2 & 0 & 1 & 2 & 0 \\ 1 & -3 & -4 & 1 & -3 \end{vmatrix}$

$= (0 + -10 + 30) - (+80 + -9 + 0)$

$= 20 - (71) = -51$

$x = \frac{34}{68} = \frac{1}{2}$

$y = \frac{-51}{68}$

$z = \frac{136}{68} = 2$

$|D_z| = \begin{vmatrix} 3 & 2 & -10 & 3 & 2 \\ 2 & 4 & 0 & 2 & 4 \\ 1 & -6 & -3 & 1 & -6 \end{vmatrix}$

$= (-36 + 0 + 120) - (-12 + 0 + -40)$

$= (84) - (-52)$

$= 136$

$\left(\frac{1}{2}, \frac{-51}{68}, 2\right)$

Math 12 Review for Matrices

14a)
$$\begin{vmatrix} 1 & 1 & -1 & 1 & 1 \\ 2 & 3 & -1 & 2 & 3 \\ -3 & -2 & 2 & -3 & -2 \end{vmatrix}$$

$(6+3+4)-(4+2+9)$

$13-(15) = -2 = |D|$

$$|D_x| = \begin{vmatrix} -1 & 1 & -1 & -1 & 1 \\ -2 & 3 & -1 & -2 & 3 \\ -3 & -2 & 2 & -3 & -2 \end{vmatrix}$$

$= (-6+3+-4)-(-4+-2+9)$

$= (-7)-(3) = -10$

$$|D_y| = \begin{vmatrix} 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & -1 & 2 & -2 \\ -3 & -3 & 2 & -3 & -3 \end{vmatrix}$$

$= (-4+-3+6)-(-4+3+-6)$

$= (-1)-(-7) = 6$

$x = \frac{-10}{2} = 5$

$y = \frac{6}{-2} = -3$

Easiest to plug in x, y solve for z.

↓

$z = 3$

$|D_z| = -6$

14b)
$$\begin{vmatrix} 6 & -9 & 4 & 6 & -9 \\ 2 & 6 & -1 & 2 & 6 \\ 4 & -3 & 2 & 4 & -3 \end{vmatrix}$$

$|D| = (72+36+-24)-(-36+18+96)$

$= (84)-(78) = 6$

$$|D_x| = \begin{vmatrix} -7 & -9 & 4 & -7 & -9 \\ 6 & 6 & -1 & 6 & 6 \\ -2 & -3 & 2 & -2 & -3 \end{vmatrix}$$

$= (-84+-18+-72)-(-108+-21+-48)$

$= (-174)-(-177) = 3$

$$|D_y| = \begin{vmatrix} 6 & -7 & 4 & 6 & -7 \\ 2 & 6 & -1 & 2 & 6 \\ 4 & -2 & 2 & 4 & -2 \end{vmatrix}$$

$= (72+28+-16)-(-28+12+96)$

$= (84)-(80) = 4$

$x = \frac{3}{6} = \frac{1}{2}$

$y = \frac{4}{6} = \frac{2}{3}$

Plug in x, y solve for z

$z = -1$

$|D_z| = \underline{-6}$