Name: Key

1) The $\lim\limits_{x \to 3} \dfrac{x^2 - 2x - 3}{x^2 - 6x + 9}$ is

A) $+\infty$
B) 0
C) nonexistent
D) $-\infty$
E) $\dfrac{2}{3}$

2) The $\lim\limits_{x \to -1} \dfrac{x^2 + 6x + 5}{x^2 - 3x - 4}$ is

A) $-\dfrac{4}{5}$
B) $\dfrac{4}{5}$
C) 1
D) $-\dfrac{3}{4}$
E) -2

3) The $\lim\limits_{x \to 3} \dfrac{x^3 - 27}{9 - x^2}$ is

A) 0
B) $\dfrac{9}{2}$
C) $-\dfrac{9}{2}$
D) nonexistent
E) 3

4) $\lim\limits_{h \to 0} \dfrac{(1+h)^3 - 1}{h}$ is

A) 0
B) nonexistent
C) 1
D) -1
E) 3

5) $\lim\limits_{x \to -2} \dfrac{|x+2|}{x+2}$ is

A) -1
B) 0
C) nonexistent
D) 1
E) none of these

6) $\lim\limits_{x \to 2} \dfrac{1 - \dfrac{4}{x} + \dfrac{4}{x^2}}{1 + \dfrac{1}{x} - \dfrac{6}{x^2}}$ is

A) 0
B) $\dfrac{1}{3}$
C) -1
D) 3
E) 1

7) $\lim\limits_{x \to 7} \dfrac{x - 7}{\sqrt{x} - \sqrt{7}}$ is

A) nonexistent
B) $2\sqrt{7}$
C) 0
D) $-2\sqrt{7}$
E) $\sqrt{7}$

8) Given function f defined below.

$$f(x) = \begin{Bmatrix} x^2 + 2x, & x \le a \\ x + 2, & x > a \end{Bmatrix}$$

Determine all values of a for which $\lim\limits_{x \to a} f(x)$ exists.

A) No value of a exists.
B) $a = -2$ and $a = 1$
C) $a = 2$ and $a = -1$
D) $a = 1$
E) $a = -2$

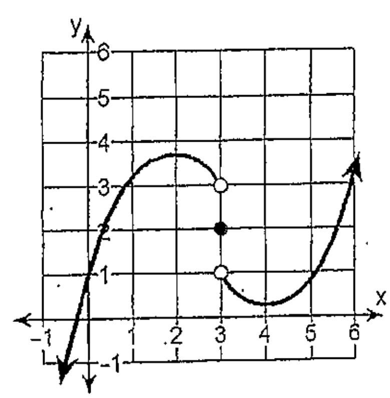
9) $\lim\limits_{x \to \infty} \dfrac{4x^2 + 16}{x^3 - 64}$ is

A) $-\dfrac{1}{4}$
B) 0
C) 4
D) ∞
E) none of these

10) $\lim\limits_{x \to -\infty} \dfrac{x^4 - 81}{3x^2 - 27}$ is

A) $-\infty$
B) 0
C) $\dfrac{1}{3}$
D) $+\infty$
E) none of these

11) The graph of function g is shown below.



Which of the following is *not* true?

A) $\lim\limits_{x \to 3^-} f(x) = 3$
B) $\lim\limits_{x \to 1} f(x) = f(1)$
C) $f(3) = 2$
D) $\lim\limits_{x \to 3} f(x)$ exists
E) $\lim\limits_{x \to 3^+} f(x) = 1$

12) $\lim_{x\to\infty} \dfrac{3-5x^2-2x^3}{6x^3+x^2-2x+1}$ is

A) $-\dfrac{1}{3}$
B) $\dfrac{1}{3}$
C) -2
D) nonexistent
E) $\dfrac{1}{2}$

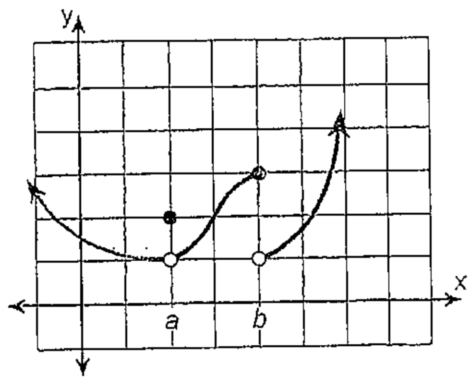
13) $\lim_{x\to\infty} \dfrac{(4-x)(4+x)}{(x+2)^2} =$

A) +1
B) 0
C) -1
D) $-\infty$
E) $+\infty$

14) If $g(x) = 2\pi^3$, then $g'(x) =$

A) $6x$
B) $6\pi^2 x$
C) $6\pi^2$
D) 0
E) 6π

15) The graph of function f is shown below.



Which of the following statements about function f are true?

I. $\lim_{x\to b} f(x)$ exists

II. $\lim_{x\to a} f(x)$ exists

III. $\lim_{x\to a} f(x) \neq f(a)$

A) *III*, only
B) *I*, *II*, and *III*
C) *II*, only
D) *I* and *II*, only
E) *II* and *III*, only

(16) Given : $f(x) = \dfrac{x}{x-1}$

Find an equation of the tangent line at $x=2$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{(x+h)}{(x+h)-1} - \frac{x}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)(x-1)-[x(x+h-1)]}{(x+h-1)(x-1)}}{h} = \lim_{h\to 0} \frac{\frac{x^2+xh-x-h-[x^2+xh-x}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+\cancel{xh}-\cancel{x}-h-\cancel{x^2}-\cancel{xh}+\cancel{x}}{(x+h-1)(x-1)} \div \frac{h}{1} = \lim_{h\to 0} \frac{-\cancel{h}}{(x+h-1)(x-1)} \cdot \frac{1}{\cancel{h}}$$

$f(2) = \frac{2}{2-1} = 2$

$(2,2)$

$$\lim_{h\to 0} \frac{-1}{(x+h-1)(x-1)} = \frac{-1}{(x-1)^2}$$

$f'(x) = \dfrac{-1}{(x-1)^2}$

$f'(2) = \dfrac{-1}{1}$

$m = -1$

$y = mx + b$

$2 = -1(2) + b$

$2 = -2 + b$

$4 = b$

$\boxed{y = -1x + 4}$

Math 12H Homework – Review

1) $\lim\limits_{x\to 3} \dfrac{x^2-2x-3}{x^2-6x+9} = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}(x-3)}$ = still D.N.E. check left, right

$f(3.1) = \dfrac{.41}{.01} = 41 = +$ $\quad f(2.9) = \dfrac{-.39}{.01} = -$ $\quad$ limit = D.N.E.

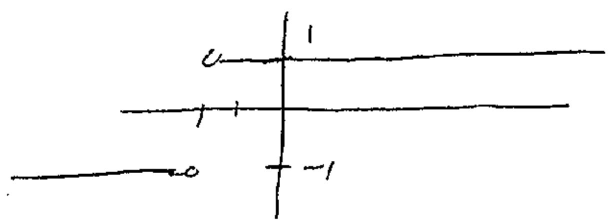
2) $\lim\limits_{x\to -1} \dfrac{x^2+6x+5}{x^2-3x-4} = \dfrac{(x+5)\cancel{(x+1)}}{(x-4)\cancel{(x+1)}} = \dfrac{-1+5}{-1-4} = \dfrac{4}{-5}$

) $\lim\limits_{x\to 3} \dfrac{x^3-27}{9-x^2} = \dfrac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(3-x)}(3+x)} = -\left(\dfrac{27}{6}\right) = -\dfrac{9}{2}$

) $\lim\limits_{h\to 0} \dfrac{(1+h)^3-1}{h} = \dfrac{(1+3h^2+3h+h^3)-1}{h} = \lim\limits_{h\to 0} \dfrac{3h^2+3h+h^3}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}(h^2+3h+3)}{\cancel{h}} = 3$

) $\lim\limits_{x\to -2} \dfrac{|x+2|}{x+2}$ = D.N.E.



) $\lim\limits_{x\to 2} \dfrac{1-\frac{4}{x}+\frac{4}{x^2}}{1+\frac{1}{x}-\frac{6}{x^2}} = \dfrac{\frac{x^2-4x+4}{x^2}}{\frac{x^2+x-6}{x^2}} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x-2)}{\cancel{x^2}} \cdot \dfrac{\cancel{x^2}}{(x+3)\cancel{(x-2)}}$

$= \lim\limits_{x\to 2} \dfrac{x-2}{x+3} = 0$

7.) $\lim_{x \to 7} \dfrac{x-7}{\sqrt{x}-\sqrt{7}} \cdot \dfrac{\sqrt{x}+\sqrt{7}}{\sqrt{x}+\sqrt{7}}$

$= \lim_{x \to 7} \dfrac{x\sqrt{x} - 7\sqrt{x} + x\sqrt{7} - 7\sqrt{7}}{x-7}$

$= \lim_{x \to 7} \dfrac{\sqrt{x}(x-7) + \sqrt{7}(x-7)}{x-7} = \lim_{x \to 7} \dfrac{\cancel{(x-7)}(\sqrt{x}+\sqrt{7})}{\cancel{x-7}} = 2$

) $f(x) = x^2+2x \quad x \leq a \quad$ left limit

$f(x) = x+2 \quad x > a \quad$ right limit

when left limit $_{x \to a}$ = right limit $_{x \to a}$ = limit $x \to a$

So $\underline{x^2+2x = x+2}$ $\quad x^2+x-2=0 \quad x=-2$ ✓

$(x+2)(x-1)=0 \quad x=+1$

) $\lim_{x \to \infty} \dfrac{4x^2+16}{x^2-64} = 0$

) $\lim_{x \to -\infty} \dfrac{x^4-81}{3x^2-27}$ biggest power on top = ∞ $\quad \dfrac{+}{+} = +$

$\dfrac{(-)^4 = +}{(-)^2 = +}$

$+\infty$

) $\boxed{D}$

Math 12 H Homework Review sheet

#12) $\lim\limits_{x\to\infty} \dfrac{3-5x^2-2x^3}{6x^3+x^2-2x+1}$

$= \lim\limits_{x\to\infty} \dfrac{-2x^3}{6x^3} = \dfrac{-2}{6} = \boxed{\dfrac{-1}{3}}$

3) $\lim\limits_{x\to\infty} \dfrac{(4-x)(4+x)}{(x+2)^2} = \dfrac{(16-x^2)}{x^2+4x+4} = \lim\limits_{x\to\infty} \dfrac{-x^2}{x^2} = \boxed{-1}$

1) $g(x) = 2\pi^3 \quad g'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{2\pi^3 - 2\pi^3}{h} = \lim\limits_{h\to 0} \dfrac{0}{h} = 0$

undefined at $h=0$ but
left limit $\to 0$
Right limit $\to 0$
So limit $= 0$

5) II is true, III is true. $\boxed{E}$

16) $F(x) = \dfrac{x}{x-1} \qquad m = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(h)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\dfrac{(x+h)}{(x+h)-1} - \left(\dfrac{x}{x-1}\right)}{h} = \dfrac{\dfrac{(x+h)(x-1)}{(x+h-1)(x-1)} - \dfrac{x(x+h-1)}{(x+h-1)(x-1)}}{h}$

$\lim\limits_{h\to 0} \dfrac{\dfrac{x^2+xh-x-h-x^2-xh+x}{(x+h-1)(x-1)}}{h} = \lim\limits_{h\to 0} \dfrac{-h}{(x+h-1)(x-1)} \cdot \dfrac{1}{h}$

$= \lim\limits_{h\to 0} \dfrac{-1}{(x+h-1)(x-1)} = \dfrac{-1}{(x-1)^2}$

$m = \dfrac{-1}{(2-1)^2} = \dfrac{-1}{1} = \boxed{-1 = m}$

$f(2) = \dfrac{2}{2-1} = 2 \qquad (2,2)$ = point on line

$(y-2) = m(x-2)$

$y-2 = -1(x-2)$

$y-2 = -x+2$
$+2 \qquad\qquad +2$

$\boxed{y = -x+4}$

17) $\displaystyle\sum_{i=1}^{n=20} \frac{2i^2+3i+4}{n}$

$$= \frac{2}{n}\sum_{i=1}^{20} i^2 + \frac{3}{n}\sum_{i=1}^{20} i + \frac{1}{n}\sum_{i=1}^{20} 4$$

$$= \frac{2}{20}\sum_{i=1}^{20} i^2 + \frac{3}{20}\sum_{i=1}^{20} i + \frac{1}{20}\sum_{i=1}^{20} 4$$

$$= \frac{2}{20}\left(\frac{2(20)^3+3(20)^2+20}{6}\right) + \frac{3}{20}\left(\frac{20(21)}{2}\right) + \frac{1}{20}\left(4(20)\right)$$

$$= \frac{1}{10}\left(\frac{16000+1200+20}{6}\right) + \frac{3}{20}\left(\frac{210}{1}\right) + \frac{1}{20}\left(\frac{80}{1}\right)$$

$$= \frac{1}{10}\left(\frac{17220}{6}\right) + 31.5 + 4$$

$$= 287 + 31.5 + 4$$

$$= \boxed{322.5}$$