Answer key - Section 8.2 - Graphs of Polar Equations pp 594-95

#2) $r = 3$ format $= r = a$ = a circle

circle of radius 3. **Graph III**

#4) $r = 1 + 2\cos\theta$ format $= a \pm b\cos\theta$ = limacon with an inner loop.
with $a < b$

Graph IV

#6) $r = \sin 4\theta$ format $r = a \sin n\theta$ 2n leaved rose
with $n = 4$ = even = 8 leaved rose

Graph V

#8) $r = 4 + 8\cos\theta$ Check symmetry around:

a) polar axis → replace θ with $-\theta$

b) pole → replace r with $-r$

c) $\theta = \frac{\pi}{2}$ → replace θ with $\pi - \theta$

polar axis

a) $r = 4 + 8\cos(-\theta) = 4 + 8\cos\theta$ = same = Symmetrical around polar axis

b) pole = replace r with $-r$

$r = 4 + 8\cos\theta \rightarrow -r = 4 + 8\cos\theta$

$= r = 4 + 8\cos(\theta + \pi)$

$= 4 + 8[\cos\theta \underbrace{\cos\pi}_{(-1)} - \sin\theta \underbrace{\sin\pi}_{(0)}]$

$= 4 + 8(-\cos\theta + 0)$

$r = 4 + 8\cos\theta \neq \quad r = 4 - 8\cos\theta$

not symmetrical around the pole

8c) Symmetrical around $\theta = \frac{\pi}{2}$ replace θ with $\pi - \theta$

$r = 4 + 8\cos\theta \rightarrow r = 4 + 8\cos(\pi - \theta)$

$= 4 + 8[\cos\pi\cos\theta + \sin\pi\sin\theta]$

$= 4 + 8[-1 \cdot \cos\theta + 0 \cdot \sin\theta]$

$= 4 - 8\cos\theta$ = not symmetrical around $\theta = \frac{\pi}{2}$

#10) $r = 5\cos\theta\csc\theta$

polar axis $\theta \rightarrow -\theta$

$5\cos(-\theta)\csc(-\theta) = 5\cos\theta(-1)\csc\theta$

$= -5\cos\theta\csc\theta$

not symmetrical around polar axis

b) pole $r \rightarrow -r$

$-r = 5\cos\theta\csc\theta$

$= 5(\cos\theta + \pi) \cdot \frac{1}{\sin(\theta+\pi)}$

$= 5(\cos\theta\cos\pi - \sin\theta\sin\pi) \cdot \frac{1}{\sin\theta\cos\pi + \cos\theta\sin\pi}$

$\cos\pi = -1$, $\sin\pi = 0$, $\cos\theta = 1$

$= 5(-\cos\theta) \cdot \frac{1}{-\sin\theta} = \frac{-5\cos\theta}{-\sin\theta} = 5\cos\theta\csc\theta$

= same ✓

Symmetrical around the pole

c) line $\theta = \frac{\pi}{2}$ $5\cos\theta\csc\theta$

$\theta \rightarrow \pi - \theta \rightarrow 5\cos(\pi-\theta)\csc(\pi-\theta)$

$= 5[\cos\pi\cos\theta + \sin\pi\sin\theta] \cdot \frac{1}{\sin\pi\cos\theta - \cos\pi\sin\theta}$

$\sin\pi = 0$, $\cos\pi = -1$

$= 5[-\cos\theta] \cdot \frac{1}{\sin\theta} = -5\cos\theta\csc\theta$ = not same

= not symmetrical around $\theta = \frac{\pi}{2}$

12) $r = \dfrac{5}{1+3\cos\theta}$ polar axis $\theta \to -\theta$

$r = \dfrac{5}{1+3\cos(-\theta)} = \dfrac{5}{1+3\cos\theta}$ = same = **Symmetrical around polar axis**

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pole $r \to -r$

$-r = \dfrac{5}{1+3\cos\theta} = r = \dfrac{5}{1+3(\cos\theta+\pi)} = \dfrac{5}{1+3[\cos\theta\cos\pi - \sin\theta\sin\pi)}$

$\cos\pi = (-1)$ $\sin\pi = (0)$

$= \dfrac{5}{1+3(-\cos\theta)} = \dfrac{5}{1-3\cos\theta}$ = **not symmetrical around the pole**

$\theta = \frac{\pi}{2}$ $\theta \to \pi - \theta$ $r = \dfrac{5}{1+3(\cos\pi-\theta)} = \dfrac{5}{1+3[\cos\pi\cos\theta + \sin\pi\sin\theta)}$

$\cos\pi = (-1)$ $\sin\pi = (0)$

$= \dfrac{5}{1+3(-\cos\theta)} = \dfrac{5}{1-3\cos\theta}$ = **not symmetrical around $\theta = \frac{\pi}{2}$**

#14) $r^2 = 9\sin\theta$ polar axis $\theta \to -\theta$

$r^2 = 9\sin(-\theta) = -9\sin\theta$ = not same **not symmetrical around polar axis**

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pole $r \to -r$ $r^2 = 9\sin\theta \to (-r)^2 = 9\sin\theta$

$= r^2 = 9\sin\theta$ = same

= **Symmetrical around pole**

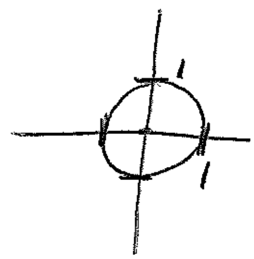
$\theta = \frac{\pi}{2}$ $\quad \theta \to \pi - \theta$

$$r^2 = 9\sin\theta \to r^2 = 9\sin(\pi - \theta)$$
$$= 9[\sin(\pi)\cos\theta - \cos\pi\sin\theta)]$$
$$\quad (0) \qquad\qquad (-1)$$
$$= 9[\sin(\theta)]$$
$$= 9\sin\theta = \text{same} =$$

Symmetrical around $\theta = \frac{\pi}{2}$

#16) $r = -1$

= Circle centered at origin with radius of 1



Calculator mode → graph = polar
angle = radian.

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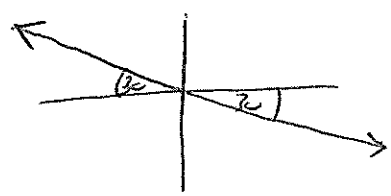
18) $\theta = \frac{5\pi}{6}$: all points whose θ coordinate $= \frac{5\pi}{6} = 150°$



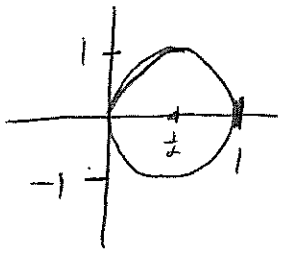
Can change to rectangular form and graph.

$$\tan\theta = \tan\frac{5\pi}{6} \to \tan\theta = -\frac{\sqrt{3}}{3} \to \frac{r\sin\theta}{r\cos\theta} = -\frac{\sqrt{3}}{3}$$

$$\to \frac{y}{x} = -\frac{\sqrt{3}}{3} \to 3y = -\sqrt{3}\,x$$

$y = -\frac{\sqrt{3}}{3}x$ in Rectangular form

#20) $r = \cos\theta$ = circle with a radius of 1



For good scaling on calculator have x window min/max at twice the values of y min/max.

x min/max $-4, 4$ y min/max $-2, 2$

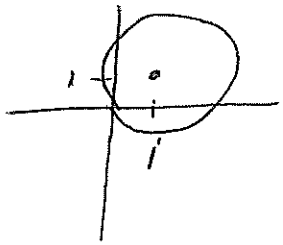
#22) $r = 2\sin\theta + 2\cos\theta$ = circle centered at $(1,1)$ with a radius of $\sqrt{2}$

$r^2 = 2r\sin\theta + 2r\cos\theta$

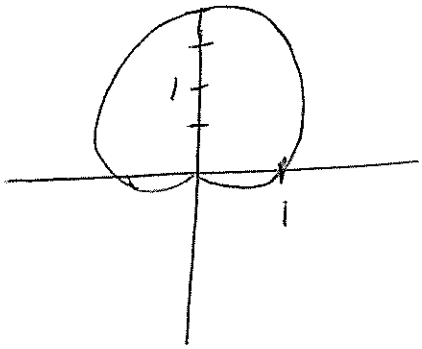
$x^2 + y^2 = 2y + 2x$

$x^2 - 2x + y^2 - 2y = 0$

$\boxed{(x-1)^2 + (y-1)^2 = 2}$



#24) $r = 1 + \sin\theta$ = Limacon where $a = b$ = cardioid



#26) $r = \cos\theta - 1$ = cardioid