

Math 12A - Answer Key - Section 9.4 - Solving Systems of

Equations using Matrices - Day 2

P 674 #30)

$$\begin{array}{rcl}
 -2x + 6y - 2z = -12 & R_1 \div -2 & 1 -3 +1 \ 6 \\
 x - 3y + 2z = 10 & & 1 -3 \ 2 \ 10 \\
 -x + 3y + 2z = 6 & & -1 \ 3 \ 2 \ 6
 \end{array}
 \begin{array}{l}
 -R_1 + R_2 \\
 R_1 + R_3
 \end{array}$$

$$\begin{array}{r}
 1 -3 \ 1 \ 6 \\
 0 \ 0 \ 1 \ 4 \\
 0 \ 0 \ 3 \ 12
 \end{array}
 \begin{array}{l}
 \\
 \\
 -3R_2 + R_3
 \end{array}
 \begin{array}{l}
 1 -3 \ 1 \ 6 \\
 0 \ 0 \ 1 \ 4 \\
 0 \ 0 \ 0 \ 0
 \end{array}
 \begin{array}{l}
 x - 3y + z = 6 \\
 z = 4 \\
 0 = 0 = \text{True}
 \end{array}$$

Many Solutions

$$x - 3y + z = 6$$

$$x = 3t + 2$$

$$(3t+2, t, 4)$$

y dropped out

$$x - 3y + y = 6$$

$$y = t$$

So let $y = t$

$$x = 3y + 2$$

$$z = 4$$

$$\begin{array}{r}
 \#34) \quad y - 5z = 7 \\
 3x + 2y = 12 \\
 3x \quad + 10z = 80
 \end{array}
 \begin{array}{l}
 \\
 \\
 -R_1 + R_2
 \end{array}
 \begin{array}{l}
 3 \ 2 \ 0 \ 12 \\
 0 \ 1 \ -5 \ 7 \\
 3 \ 0 \ 10 \ 80
 \end{array}
 \begin{array}{l}
 \\
 \\
 -R_1 + R_3
 \end{array}
 \begin{array}{l}
 3 \ 2 \ 0 \ 12 \\
 0 \ 1 \ -5 \ 7 \\
 0 \ -2 \ 10 \ 68
 \end{array}
 \begin{array}{l}
 \\
 \\
 2R_2 + R_3
 \end{array}$$

$$\begin{array}{r}
 3 \ 2 \ 0 \ 12 \\
 0 \ 1 \ -5 \ 7 \\
 0 \ 0 \ 0 \ 82
 \end{array}
 \begin{array}{l}
 3x + 2y = 12 \\
 y - 5z = 7 \\
 0 = 82 \rightarrow \text{False} = \underline{\text{no Solution}}
 \end{array}$$

inconsistent

$$\#38) \begin{cases} 3x - y + 2z = -1 \\ 4x - 2y + z = -7 \\ -x + 3y - 2z = -1 \end{cases} \rightarrow \begin{bmatrix} -1 & 3 & -2 & -1 \\ 4 & -2 & 1 & -7 \\ 3 & -1 & 2 & -1 \end{bmatrix} \begin{matrix} 4R_1 + R_2 \\ 3R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} -1 & 3 & -2 & -1 \\ 0 & 10 & -7 & -11 \\ 0 & 8 & -4 & -4 \end{bmatrix} \begin{matrix} R_1 \div -1 \\ 4R_2 \\ -5R_3 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -28 & -44 \\ 0 & -40 & 20 & 20 \end{bmatrix} \begin{matrix} R_1 \div -1 \\ R_2 \div 4 \\ R_2 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -7 & -11 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} -2R_3 + R_1 \\ 7R_3 + R_2 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 & -5 \\ 0 & 10 & 0 & 10 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} R_3 \div 10 \\ 3R_3 + R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} x = -2 \\ y = 1 \\ z = 3 \end{matrix} \quad (-2, 1, 3)$$

$$\#42) \begin{cases} x - 3y + 2z + w = -2 \\ x - 2y \\ 3x + 2z + w = -3 \end{cases} \rightarrow \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 1 & -2 & 0 & -2 & -10 \\ 3 & 0 & 2 & 1 & -3 \end{bmatrix} \begin{matrix} -2w = -10 \\ z + 5w = 15 \\ -3R_1 + R_3 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \end{bmatrix} \begin{matrix} -R_1 + R_2 \\ -3R_1 + R_3 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \end{bmatrix} \begin{matrix} -9R_3 + R_4 \end{matrix}$$

$$\begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \\ 0 & 0 & 14 & 25 & 75 \end{bmatrix} \begin{matrix} -14R_3 + R_4 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \\ 0 & 0 & 0 & -45 & -135 \end{bmatrix} \begin{matrix} R_4 \div -45 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 & 1 & -2 \\ 0 & 1 & -2 & -3 & -8 \\ 0 & 0 & 1 & 5 & 15 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} -1R_4 + R_1 \\ 2R_4 + R_2 \\ -5R_4 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -3 & 2 & 0 & -5 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} -2R_3 + R_1 \\ 2R_3 + R_2 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} 3R_3 + R_1 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} x = -2 \\ y = 1 \\ z = 0 \\ w = 3 \end{matrix} \rightarrow (-2, 1, 0, 3)$$