Math 12H - Answer Key — Inverses - Section 9.6 pp. 697-98

#4) Show A and B are Inverses by proving $AB = BA = I$.

$$\underset{A}{\begin{bmatrix} 3 & 2 & 4 \\ 1 & 1 & -6 \\ 2 & 1 & 12 \end{bmatrix}} \cdot \underset{B}{\begin{bmatrix} 9 & -10 & -8 \\ -12 & 14 & 11 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}} = \begin{bmatrix} 27-24-2 & -30+28+2 & -24+22+2 \\ 9-12+3 & -10+14-3 & -8+11-3 \\ 18-12-6 & -20+14+6 & -16+11+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underset{B}{\begin{bmatrix} 9 & -10 & -8 \\ -12 & 14 & 11 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}} \cdot \underset{A}{\begin{bmatrix} 3 & 2 & 4 \\ 1 & 1 & -6 \\ 2 & 1 & 12 \end{bmatrix}} = \begin{bmatrix} 27-10-16 & 18-10-8 & 36+60-96 \\ -36+14+22 & -24+14+11 & -48-84+132 \\ -1.5+.5+1 & -1+\frac{1}{2}+\frac{1}{2} & -2-3+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A \cdot B = B \cdot A = I$ Therefore $A = B^{-1}$ or $A^{-1} = B$

#6) B

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ -2 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{2R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 0 & 5 & 4 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \div 2} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 5 & 4 & 2 & 0 & 1 \end{array}\right] -5R_2+R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 & 2 & -2.5 & 1 \end{array}\right] \begin{matrix} -3R_2+R_1 \\ \\ R_3 \div -1 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1.5 & 0 \\ 0 & 1 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 & 2.5 & -1 \end{array}\right] \begin{matrix} R_3+R_1 \\ -R_3+R_2 \\ \\ \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & -2 & 2.5 & -1 \end{array}\right]$$

$$B^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ 2 & -2 & 1 \\ -2 & 2.5 & -1 \end{bmatrix}$$

$$B \cdot B^{-1} = B^{-1} \cdot B = I \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ✓$$

#8) $A = \begin{bmatrix} 3 & 4 \\ 7 & 9 \end{bmatrix}$ $A^{-1} = \frac{1}{|A|} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ad-bc

$|A| = 3(9) - 4(7)$

$27 - 28 = -1$

$A^{-1} = \frac{1}{-1} \begin{bmatrix} 9 & -4 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -9 & 4 \\ 7 & -3 \end{bmatrix}$

$A \cdot A^{-1} = A^{-1} \cdot A = I$ ✓

#10) $\begin{bmatrix} -7 & 4 \\ 8 & -5 \end{bmatrix}$ $|A| = (-7)(-5) - (4)(8)$

$= 35 - 32 = 3$

$A^{-1} = \frac{1}{3} \begin{bmatrix} -5 & -4 \\ -8 & -7 \end{bmatrix} = \begin{bmatrix} -\frac{5}{3} & -\frac{4}{3} \\ -\frac{8}{3} & -\frac{7}{3} \end{bmatrix}$

#12) $\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ 5 & 4 \end{bmatrix}$ $|A| = (\frac{1}{2})(4) - (\frac{1}{3})(5)$

$= 2 - \frac{5}{3} = \frac{1}{3}$

$A^{-1} = \frac{1}{(\frac{1}{3})} \cdot \begin{bmatrix} 4 & -\frac{1}{3} \\ -5 & \frac{1}{2} \end{bmatrix} = 3 \begin{bmatrix} 4 & -\frac{1}{3} \\ -5 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 12 & -1 \\ -15 & 1.5 \end{bmatrix}$

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#16)
$$\left[\begin{array}{ccc|ccc} 5 & 7 & 4 & 1 & 0 & 0 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 6 & 7 & 5 & 0 & 0 & 1 \end{array}\right] \; R_3 - R_1 \; \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 0 & 1 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 6 & 7 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -3R_1 + R_2 \\ -6R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & -1 & 0 & 3 & 1 & -3 \\ 0 & 7 & -1 & 6 & 0 & -5 \end{array}\right] \; 7R_2 + R_3 \; \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & -1 & 0 & 3 & 1 & -3 \\ 0 & 0 & -1 & 27 & 7 & -26 \end{array}\right] \begin{array}{l} R_1 + R_3 \\ R_2 \div -1 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 26 & 7 & -25 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & -1 & 27 & 7 & -26 \end{array}\right] \; R_3 \div -1 \; \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 26 & 7 & -25 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{array}\right]$$

$$\text{Inverse} = \begin{bmatrix} 26 & 7 & -25 \\ -3 & -1 & 3 \\ -27 & -7 & 26 \end{bmatrix}$$