Math 12H - Answer Key - Section 9.7 - Matrix Applications pp 713-14

#30) $6x + 12y = 33$
$4x + 7y = 20$

$|A| = \begin{vmatrix} 6 & 12 \\ 4 & 7 \end{vmatrix} = 42 - 48 = -6$

$|A_x| = \begin{vmatrix} 33 & 12 \\ 20 & 17 \end{vmatrix} = 231 - 240 = -9$

$x = \frac{|A_x|}{|A|} = \frac{-9}{-6} = 1.5$

$|A_y| = \begin{vmatrix} 6 & 33 \\ 4 & 20 \end{vmatrix} = 120 - 132 = -12$

$y = \frac{|A_y|}{|A|} = \frac{-12}{-6} = 2$

$\boxed{x = 1.5 \quad y = 2}$

#34) $10x - 17y = 21$
$20x - 31y = 39$

$|A| = \begin{vmatrix} 10 & -17 \\ 20 & -31 \end{vmatrix} = -310 - (-340) = 30$

$|A_x| = \begin{vmatrix} 21 & -17 \\ 39 & -31 \end{vmatrix} = -651 - (-663) = 12$

$x = \frac{|A_x|}{|A|} = \frac{12}{30} = \frac{2}{5}$

$|A_y| = \begin{vmatrix} 10 & 21 \\ 20 & 39 \end{vmatrix} = 390 - 420 = -30$

$y = \frac{|A_y|}{|A|} = \frac{-30}{30} = -1$

$\boxed{x = \frac{2}{5} \quad y = -1}$

#38) $-2a + c = 2$
$a + 2b - c = 9$
$3a + 5b + 2c = 22$

$|A| = \begin{vmatrix} -2 & 0 & 1 & -2 & 0 \\ 1 & 2 & -1 & 1 & 2 \\ 3 & 5 & 2 & 3 & 5 \end{vmatrix}$

$(-8 + 0 + 5) - (0 + 10 + 6) = -19$
$-3 - 16$

$|A_x| = \begin{vmatrix} 2 & 0 & 1 & 2 & 0 \\ 9 & 2 & -1 & 9 & 2 \\ 22 & 5 & 2 & 22 & 5 \end{vmatrix}$

$= (8 + 0 + 45) - (0 + -10 + 44) = 19$
$53 - (34)$

$|A_c| = \begin{vmatrix} -2 & 0 & 2 & -2 & 0 \\ 1 & 2 & 9 & 1 & 2 \\ 3 & 5 & 22 & 3 & 5 \end{vmatrix}$

$(-88 + 0 + 10) - (0 + -90 + 12) = 0$
$(-78) - (-78)$

$|A_b| = \begin{vmatrix} -2 & 2 & 1 & -2 & 2 \\ 1 & 9 & -1 & 1 & 9 \\ 3 & 22 & 2 & 3 & 22 \end{vmatrix}$

$(-36 + -6 + 22) - (4 + 44 + 27) = -95$
$-20 - (75)$

$a = \frac{|A_x|}{|A|} = \frac{19}{-19} = -1 = a$

$b = \frac{|A_b|}{|A|} = \frac{-95}{-19} = 5 = b$

$c = \frac{|A_c|}{|A|} = \frac{0}{-19} = 0 = c$

#40) $2x - y = 5$

$5x + 3z = 19$

$4x + 7z = 17$

$$|A| = \begin{vmatrix} 2 & -1 & 0 & 2 & -1 \\ 5 & 0 & 3 & 5 & 0 \\ 0 & 4 & 7 & 0 & 4 \end{vmatrix}$$

$(0+0+0) - (-35+24+0) = \underline{\underline{11}}$

$0 - (-11)$

$$|Ax| = \begin{vmatrix} 5 & -1 & 0 & 5 & -1 \\ 19 & 0 & 3 & 19 & 0 \\ 17 & 4 & 7 & 17 & 4 \end{vmatrix}$$

$(0 - 51 + 0) - (-133 + 60 + 0) = \underline{\underline{22}}$

$(-51) - (-73)$

$$|Ay| = \begin{vmatrix} 2 & 5 & 0 & 2 & 5 \\ 5 & 19 & 3 & 5 & 19 \\ 0 & 17 & 7 & 0 & 17 \end{vmatrix}$$

$(266 + 0 + 0) - (175 + 102 + 0)$

$(266) - (277) = \underline{\underline{-11}}$

$$|Az| = \begin{vmatrix} 2 & -1 & 5 & 2 & -1 \\ 5 & 0 & 19 & 5 & 0 \\ 0 & 4 & 17 & 0 & 4 \end{vmatrix}$$

$(0 + 0 + 100) - (-85 + 152 + 0)$

$100 - (67) = 33$

$x = \frac{|Ax|}{|A|} = \frac{22}{11} = 2 = x$

$y = \frac{|Ay|}{|A|} = \frac{-11}{11} = -1 = y$

$z = \frac{|Az|}{|A|} = \frac{33}{11} = 3 = z$

#42) $2x - 5y = 4$

$x + y - z = 8$

$3x + 5z = 0$

$$|Ax| = \begin{vmatrix} 4 & -5 & 0 & 4 & -5 \\ 8 & 1 & -1 & 8 & 1 \\ 0 & 0 & 5 & 0 & 0 \end{vmatrix}$$

$= (20 + 0 + 0) - (-200 + 0 + 0) = \underline{\underline{220}}$

$$|A| = \begin{vmatrix} 2 & -5 & 0 & 2 & -5 \\ 1 & 1 & -1 & 1 & 1 \\ 3 & 0 & 5 & 3 & 0 \end{vmatrix}$$

$(10 + 15 + 0) - (-25 + 0 + 0)$

$= 25 - -25 = \underline{\underline{50}}$

$$|Ay| = \begin{vmatrix} 2 & 4 & 0 & 2 & 4 \\ 1 & 8 & -1 & 1 & 8 \\ 3 & 0 & 5 & 3 & 0 \end{vmatrix}$$

$(80 + -12 + 0) - (20 + 0 + 0) = 48$

$$|Az| = \begin{vmatrix} 2 & -5 & 4 & 2 & -5 \\ 1 & 1 & 8 & 1 & 1 \\ 3 & 0 & 0 & 3 & 0 \end{vmatrix}$$

$(0 + -120 + 0) - (0 + 0 + 12) = -132$

$x = \frac{|Ax|}{|A|} = \frac{220}{50} = 4.2 = \frac{22}{5} = x$

$y = \frac{|Ay|}{|A|} = \frac{48}{50} = \frac{24}{25} = \frac{24}{25} = y$

$z = \frac{|Az|}{|A|} = \frac{-132}{50} = -\frac{66}{25} = z$

48) Solve for x

$$\begin{vmatrix} x & 1 & 1 \\ 1 & 1 & x \\ x & 1 & x \end{vmatrix} = 0$$

$$|A| = \begin{vmatrix} x & 1 & 1 & x & 1 \\ 1 & 1 & x & 1 & 1 \\ x & 1 & x & x & 1 \end{vmatrix} = (x^2 + x^2 + 1) - (x + x^2 + x) = 0$$

$$2x^2 + 1 - (x^2 + 2x) = 0$$

$$2x^2 + 1 - x^2 - 2x = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0 \qquad \boxed{x = 1}$$

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52) Triangle has vertices of

(1, 0) (3, 5) (−2, 2)

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 3 & 5 & 1 \\ -2 & 2 & 1 \end{vmatrix} = \pm\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 3 & 5 & 1 & 3 & 5 \\ -2 & 2 & 1 & -2 & 2 \end{vmatrix}$$

(x's y's)

$$= \pm\frac{1}{2}\left[(5 + 0 + 6) - (0 + 2 + -10)\right]$$

$$= \pm\frac{1}{2}\left[(11 - -8)\right]$$

$$= \frac{1}{2}(19) = \frac{19}{2} = \boxed{9.5}$$