Math 12 H Lesson Plans Section 9.8 Answer Key

Partial Fractions

H.W p. 720 # 2) $\frac{x}{x^2+3x-4} = \frac{x}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$

#12) $\frac{2x}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$ multiply all by L.C.D.

$\frac{2x\ (x+1)(x-1)}{(x+1)(x-1)} = \frac{A(x+1)(x-1)}{(x-1)} + \frac{B(x+1)(x-1)}{x+1}$

$2x = A(x+1) + B(x-1) = Ax + A + Bx - B$

$2x = (A+B)x + A - B \rightarrow \begin{cases} A+B = 2 \\ A-B = 0 \end{cases}$

$2A = 2$

$A = 1, \quad B = 1$

So $\frac{2x}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{1}{x+1}$

L.C.D = $x(x+3)$

14) $\frac{x+6}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$

$\frac{(x+6)\cdot x(x+3)}{x(x+3)} = \frac{A(x)(x+3)}{x} + \frac{B(x)(x+3)}{x+3}$

$x+6 = A(x+3) + B(x) = Ax + 3A + Bx$

$x+6 = (A+B)(x) + 3A$

$\rightarrow \begin{cases} A+B = 1 \\ 3A = 6 \end{cases}$

$A = 2, \quad B = -1$

$\frac{x+6}{x(x+3)} = \frac{2}{x} - \frac{1}{x+3}$

16) $\frac{x-12}{x^2-4x} = \frac{x-12}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$ L.C.D = $x(x-4)$

$x-12 = A(x-4) + B(x)$

$x-12 = Ax - 4A + Bx$

$x-12 = (A+B)x - 4A \rightarrow \begin{cases} A+B=1 \\ -12=-4A \end{cases}$

$A=3,\ B=-2$

$\frac{x-12}{x^2-4x} = \frac{3}{x} - \frac{2}{x-4}$

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18) $\frac{2x+1}{x^2+x-2} = \frac{2x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$ L.C.D = $(x+2)(x-1)$

$2x+1 = A(x-1) + B(x+2) = Ax - A + Bx + 2B$

$2x+1 = (A+B)(x) - A + 2B$

$\rightarrow \begin{cases} A+B=2 \\ -A+2B=1 \end{cases}$

$3B = 3 \qquad B=1,\ A=1$

$\frac{2x+1}{x^2+x-2} = \frac{1}{x+2} + \frac{1}{x-1}$

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20) $\frac{7x-3}{x^3+2x^2-3x} = \frac{7x-3}{x(x+3)(x-1)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-1} = \boxed{\frac{1}{x} - \frac{2}{x+3} + \frac{1}{x-1}}$

$\frac{7x-3}{x(x+3)(x-1)} = \frac{A\,x(x+3)(x-1)}{x} + \frac{B(x)(x+3)(x-1)}{x+3} + \frac{C(x)(x+3)(x-1)}{x-1}$

$A=1$
$B=-2$
$C=1$

$7x-3 = A(x+3)(x-1) + B(x)(x-1) + C(x)(x+3)$

$7x-3 = A(x^2+2x-3) + B(x^2-x) + C(x^2+3x)$

$7x-3 = Ax^2 + 2Ax - 3A + Bx^2 - Bx + Cx^2 + 3Cx$

$7x-3 = (A+B+C)x^2 + (2A-B+3C)x - 3A$

$\begin{cases} A+B+C=0 \\ 2A-B+3C=7 \\ -3A=-3 \leftarrow A=1 \end{cases}$

$1+B+C=0 \quad = \quad B+C=-1$
$2-B+3C=7 \quad = \quad -B+3C=5$
$4C=4$

$C=1$ so $B=-2$