Partial Fraction Assignment - Answer Key - Day 2 section 9.8

p 720 #24) $\dfrac{-3x^2-3x+27}{(x+2)(2x^2+3x-9)} = \dfrac{-3x^2-3x+27}{(x+2)(2x-3)(x+3)}$

$= \dfrac{-3x^2-3x+27}{(x+2)(2x-3)(x+3)} = \dfrac{A}{x+2} + \dfrac{B}{2x-3} + \dfrac{C}{x+3}$

$-3x^2-3x+27 = A(2x-3)(x+3) + B(x+2)(x+3) + C(x+2)(2x-3)$

$-3x^2-3x+27 = A(2x^2+3x-9) + B(x^2+5x+6) + C(2x^2+x-6)$

$-3x^2-3x+27 = 2Ax^2+3Ax-9A + Bx^2+5Bx+6B + 2Cx^2+Cx-6C$

$-3x^2-3x+27 = (2A+B+2C)x^2 + (3A+5B+C)x - 9A+6B-6C$

$2A+B+2C = -3$
$3A+5B+C = -3$
$-9A+6B-6C = 27$

$\det = 63$
$|D_A| = -189$
$|D_B| = 63$
$|D_C| = 63$

$A = -3$
$B = 1$
$C = 1$

$-3x^3-3x+27 = \dfrac{-3}{x+2} + \dfrac{1}{2x-3} + \dfrac{1}{x+3}$

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26) $\dfrac{3x^2+5x-13}{(3x+2)(x^2-4x+4)} = \dfrac{3x^2+5x-13}{(3x+2)(x-2)^2}$

L.C.D $= (3x+2)(x-2)^2$

$\dfrac{3x^2+5x-13}{(3x+2)(x-2)^2} = \dfrac{A}{3x+2} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$

$3x^2+5x-13 = A(x-2)^2 + B(3x+2)(x-2) + C(3x+2)$

$3x^2+5x-13 = A(x^2-4x+4) + B(3x^2-4x-4) + C(3x+2)$

$3x^2+5x-13 = Ax^2-4Ax+4A + 3Bx^2-4Bx-4B + 3Cx+2C$

$3x^2+5x-13 = (A+3B)x^2 + (-4A-4B+3C)x + 4A-4B+2C$

$A+3B = 3$
$-4A-4B+3C = 5$
$4A-4B+2C = -13$

$$A + 3B = 3$$
$$-4A - 4B + 3C = 5$$
$$4A - 4B + 2C = -13$$

$|D| = 64$

$|D_A| = -135$

$|D_B| = 109$

$|D_C| = 72$

$A = \frac{-135}{64} \quad B = \frac{109}{64} \quad C = \frac{72}{64} = \frac{9}{8}$

$$\frac{3x^2+5x-13}{(3x+2)(x-2)^2} = \frac{\frac{-135}{64}}{3x+2} + \frac{\frac{109}{64}}{x-2} + \frac{\frac{9}{8}}{(x-2)^2}$$

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28) $$\frac{x-4}{(2x-5)^2} = \frac{A}{(2x-5)} + \frac{B}{(2x-5)^2}$$

$$x-4 = A(2x-5) + B$$
$$x-4 = 2Ax - 5A + B$$
$$x-4 = (2A)x - 5A + B$$

$$\left.\begin{aligned} 2A &= 1 \\ -5A + B &= -4 \end{aligned}\right] \rightarrow \quad A = \frac{1}{2}, \quad B = -1.5 = -\frac{3}{2}$$

$$\frac{x-4}{(2x-5)^2} = \frac{\frac{1}{2}}{2x-5} - \frac{\frac{3}{2}}{(2x-5)^2}$$

#32)

$$\frac{-2x^2+5x-1}{x^4-2x^3+2x-1} = \frac{-2x^2+5x-1}{(x-1)(x^3-x^2-x+1)} = \frac{-2x^2+5x-1}{(x+1)(x-1)^3}$$

$$\frac{-2x^2+5x-1}{(x-1)^3(x+1)} = \frac{A}{x+1} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$$

$$-2x^2+5x-1 = A(x-1)^3 + B(x+1)(x-1)^2 + C(x+1)(x-1) + D(x+1)$$

$$= A(x^3-3x^2+3x-1) + B(x+1)(x-1)^2 + C(x^2-1) + D(x+1)$$

$$= A(x^3-3x^2+3x-1) + B(x^3-x^2-x+1) + C(x^2-1) + D(x+1)$$

$$-2x^2+5x+1 = Ax^3-3Ax^2+3Ax-A+Bx^3-Bx^2-Bx+B+Cx^2-C+Dx+D$$

$$-2x^2+5x+1 = (A+B)x^3 + (-3A-B+C)x^2 + (3A-B+D)x + (-A+B-C+D)$$

$$\begin{aligned} A+B &= 0 \\ -3A-B+C &= -2 \\ 3A-B+D &= 5 \\ -A+B-C+D &= -1 \end{aligned}$$

$$\begin{aligned} A+B &= 0 \\ 2B+C &= -2 \\ -2B+C+D &= 3 \\ 2B-3C+4D &= 2 \end{aligned}$$

$$\begin{aligned} A+B &= 0 \\ 2B+C &= -2 \\ 2C+D &= 1 \\ -3C+5D &= 5 \end{aligned}$$

$$\begin{aligned} A+B &= 0 \\ 2B+C &= -2 \\ 2C+D &= 1 \\ 6D &= 6 \end{aligned}$$

$$\begin{aligned} D &= 1 \\ C &= 0 \\ B &= -1 \\ A &= 1 \end{aligned}$$

$$\frac{-2x^2+5x-1}{(x-1)^3(x+1)} = \frac{1}{x+1} - \frac{1}{x-1} + \frac{0}{(x-1)^2} + \frac{1}{(x-1)^3}$$

36) $$\frac{3x^2-2x+8}{x^3-x^2+2x-2} = \frac{3x^2-2x+8}{x^2(x-1)+2(x-1)} = \frac{3x^2-2x+8}{(x^2+2)(x-1)}$$

$$\frac{3x^2-2x+8}{(x^2+2)(x-1)} = \frac{Ax+B}{(x^2+2)} + \frac{C}{(x-1)}$$

$$3x^2-2x+8 = (Ax+B)(x-1) + C(x^2+2)$$

$$3x^2-2x+8 = Ax^2+Bx-Ax-B+Cx^2+2C$$

$$3x^2-2x+8 = (A+C)x^2+(-A+B)(x)-B+2C$$

$$\begin{aligned} A \quad\quad +C &= 3 \\ -A+B &= -2 \\ -B+2C &= 8 \end{aligned}$$

$$\begin{aligned} A \quad\quad +C &= 3 \\ B+C &= 1 \\ 2C &= 6 \end{aligned}$$

$$\begin{aligned} A &= 0 \\ B &= -2 \\ C &= 3 \end{aligned}$$

$$\frac{3x^2-2x+8}{(x^2+2)(x-1)} = \frac{0x-2}{x^2+2} + \frac{3}{x-1} = \frac{-2}{x^2+2} + \frac{3}{x-1}$$