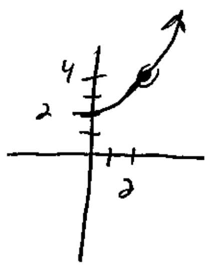
Math 12 H Lesson Plan - Section 12.1 Finding limits Graphically and numerically.

Limit of a function - What is happening to the values $f(x)$ (y) as x approaches a number c.

Ex: $f(x) = x^2 - x + 2$ What happens to $f(x)$ (y) as x approaches the number 2?



Write $\lim_{x \to a} f(x) = L$

the limit of $f(x)$ as x approaches the number a equals the value L.

We are talking about taking x "sufficiently" close to a, but <u>not</u> equal to a.

In this section we will use graphs and tables from the calculator to estimate limits of functions.

Ex: $\lim_{x \to 1} \frac{x-1}{x^2-1}$

Note that the function $f(x) = \frac{x-1}{x^2-1}$ is not defined at $x=1$, but it doesn't mean we can't have a limit as x <u>approaches</u> 1.

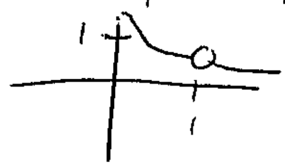
If you look closely at the graph, there is a hole or point of "discontinuity" at $x=1$.
However on either side of 1, what is y approaching as x gets very close to 1.?

Adjust Δ tbl on calculator in tbl set and evaluate.

for $x<1$		for $x>1$	
$x=.5$	$y=.666\breve{6}$	$x=1.5$	$y=.4000$
$x=.9$	$y=.526316$	$x=1.1$	$y=.476190$
$x=.99$	$y=.502513$	$x=1.01$	$y=.497512$
$x=.999$	$y=.500250$	$x=1.001$	$y=.499750$
$x=.9999$	$y=.500025$	$x=1.0001$	$y=.499975$

On this basis as x gets closer to 1 from either the left side or the right side, y is approaching $\frac{1}{2}$

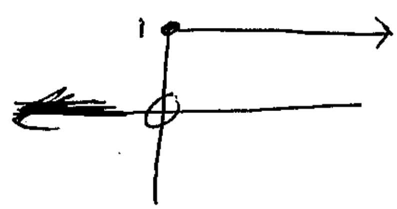
So: $\lim\limits_{x\to 1} \dfrac{x-1}{x^2-1} = \underline{\underline{\dfrac{1}{2}}}$



gap at $x=\frac{1}{2}$

Ex: limits that do not exist

$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & , \text{f } t \geq 0 \end{cases}$$



$\lim\limits_{t\to 0} H(t) =$ does not exist. Left side limit is different from right side limit.

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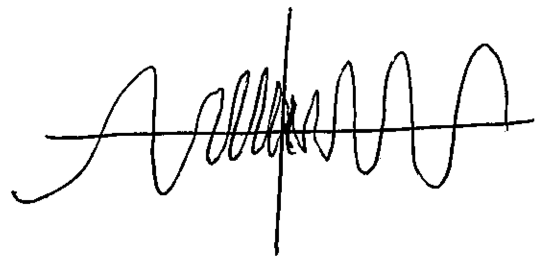
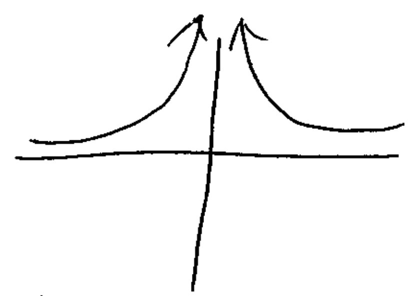
Ex: $\lim_{x \to 0} \sin \frac{\pi}{x} = D.N.E.$



Depending on values you check, the limit might look like 0. However, the graph shows that the function oscilates between −1 and 1 infinitely as x approaches 0. ∴ limit d.n.e.

Ex: $\lim_{x \to 0} \frac{1}{x^2} = D.N.E.$



more accurate would be

$\lim_{x \to 0} \frac{1}{x^2} = +\infty.$ It is still not an actual limit value, but it is more accurate than D.N.E.

Ex: Left Hand and Right Hand limits

The limit of f(x) as x approaches from the left:

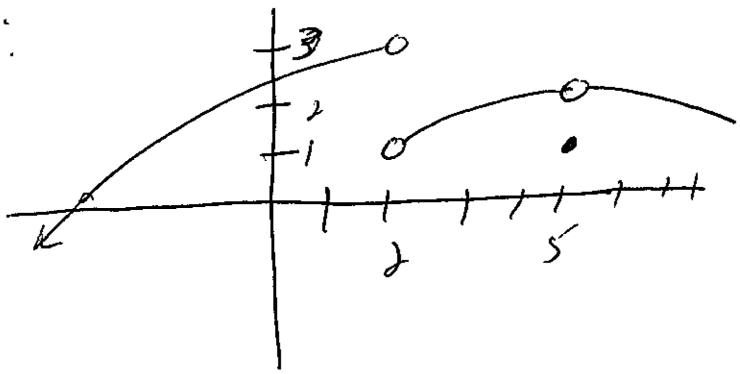
$\lim_{x \to a^-} f(x) = L_1$

The limit of f(x) as x approaches from the right:

$\lim_{x \to a^+} f(x) = L_2$

If $L_1 = L_2$ then f(x) has a limit as $x \to a$...

Ex:



$\lim_{x \to 2^-} f(x) = 3$ $\lim_{x \to 2^+} f(x) = 1$ $f(2) = \text{undefined}$

Since left hand limit $\neq$ right hand limit

then $\lim_{x \to 2} f(x) = \text{D.N.E.}$

$\lim_{x \to 5^-} f(x) = 2$ $\lim_{x \to 5^+} f(x) = 2$ $f(5) = 1$