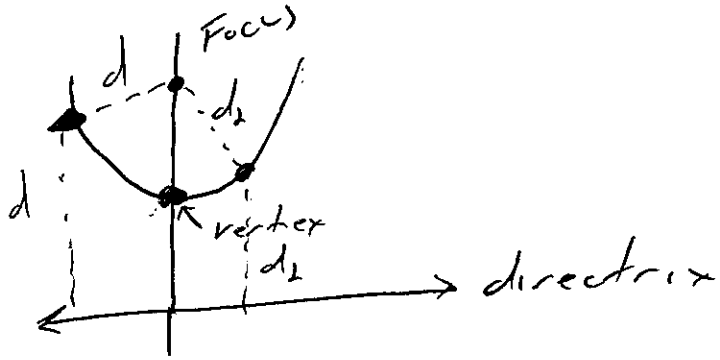
Math 12 H Lesson Plan - Section 10.1 Parabolas

Geometric Definition of a parabola = set of all points equidistant from a fixed point F, (Focus) and a fixed line L called the directrix.

Ex:



all points on parabola are same distance from focus and directrix.

The Vertex is halfway between the Focus and directrix.

The axis of symmetry runs through the Focus and is ⊥ to the directrix.

For vertices at the origin, there will be either a
- horizontal axis of symmetry = $y=0$ or
- vertical axis of symmetry = $x=0$.

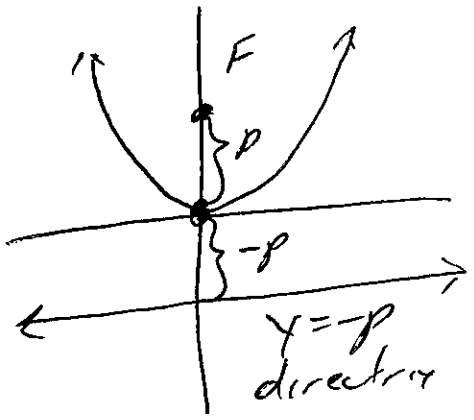
Generic Formulas p 745 and 747.

For a vertex of (0,0) with a vertical axis of symmetry

Focus is at point $(0,p)$ $F(0,p)$

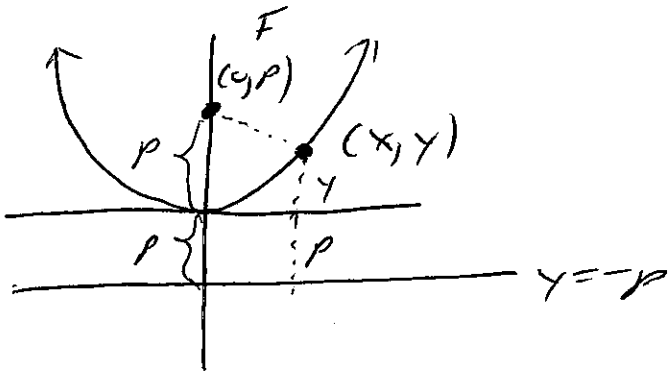
Directrix has the equation $y=-p$

where p is the distance between either focus and origin (vertex) or directrix and vertex.



Standard Equation = $\boxed{x^2=4py}$

Proof of basic Equation



Proof of equation, if point is equally distant from focus and directrix.

Distance of (x,y) from Focus $(0,p)$

$$= \sqrt{(x-0)^2+(y-p)^2} = \sqrt{x^2+(y-p)^2}$$

Distance from (x,y) to Directrix $= |y+p|$

So $\sqrt{x^2+(y-p)^2} = |y+p|$.

$$\left(\sqrt{x^2+(y-p)^2}\right)^2 = (y+p)^2 = x^2+(y-p)^2 = (y+p)^2$$

$$x^2+y^2-2py+p^2 = y^2+2py+p^2 = \boxed{x^2=4py}$$

when $p>0$ parabola opens up.

If $p<0$ parabola opens down.

Parabola with a vertical axis Vertex at (0,0)

Vertex $V(0,0)$ Graph $x^2 = 4py$

Focus $F(0,p)$

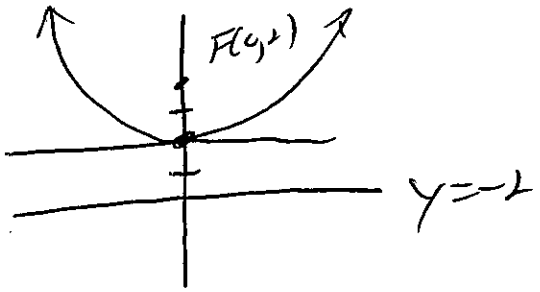
Directrix $y = -p$

Ex: Find an equation of a parabola with $V(0,0)$ $F(0,2)$ and sketch.

$F(0,2) = p = 2$ So directrix is $y = -2$

Equation $x^2 = 4py$ $x^2 = 4(2)(y)$ $x^2 = 8y$

$= y = \frac{x^2}{8}$



$p > 0$

So parabola opens up.

Ex: Find the focus and directrix of the parabola $y = -x^2$ and sketch.

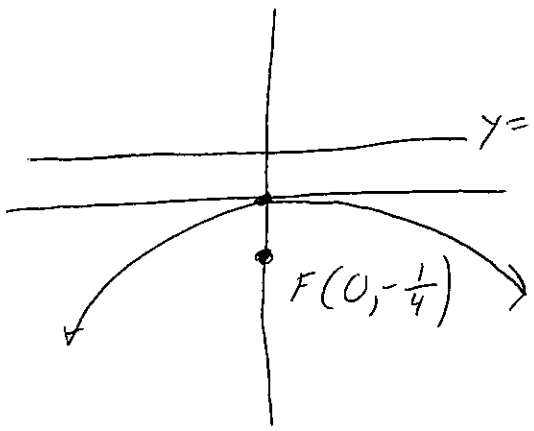
$x^2 = 4py$ $y = -x^2$ → $x^2 = -y$ $x^2 = +4py$

So $4p = -1$ $p = -\frac{1}{4}$

Focus $= \left(0, -\frac{1}{4}\right)$

Directrix $y = -p$

$= y = \frac{1}{4}$



Parabola with a horizontal axis of symmetry $y = 0$

Reflects parabola through the line $y = x$ (Swap x and y)

$x^2 = 4py$

Vertex $V = (0,0)$

Focus $(0, p)$ y-value on focus

Directrix $y = -p$

$\rightarrow$

$y^2 = 4px$

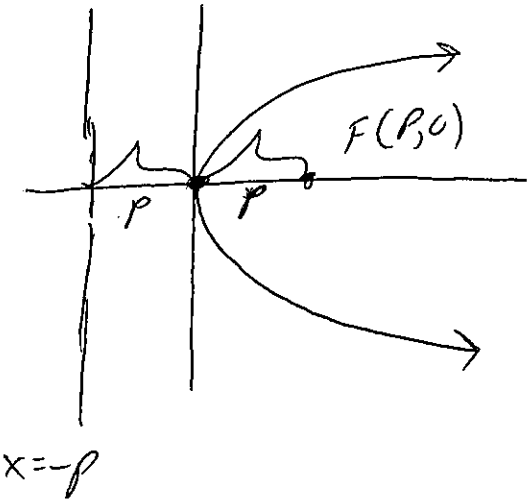
Vertex $V(0,0)$

Focus $(p, 0)$ x-value on focus

Directrix $x = -p$

$p > 0$ = opens right

$p < 0$ = opens left.



Ex: A Parabola has an equation of $6x + y^2 = 0$

Find the Focus and Directrix and sketch the graph.

Standard form $y^2 = 4px$

$y^2 = -6x$

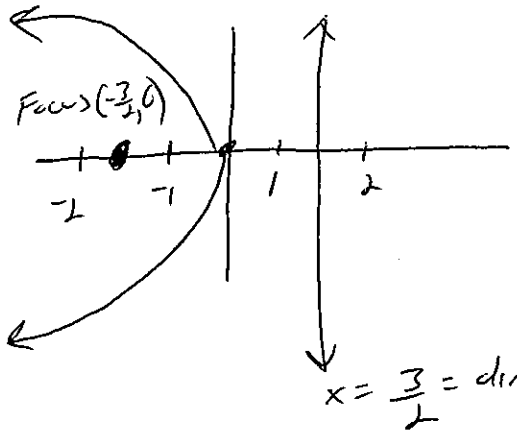
& $4p = -6$

$p = \frac{-6}{4} = \frac{-3}{2}$

$p = \frac{-3}{2}$ so $\boxed{\text{Focus} = \left(\frac{-3}{2}, 0\right)}$

Directrix $x = -p$

$\boxed{x = \frac{3}{2}}$



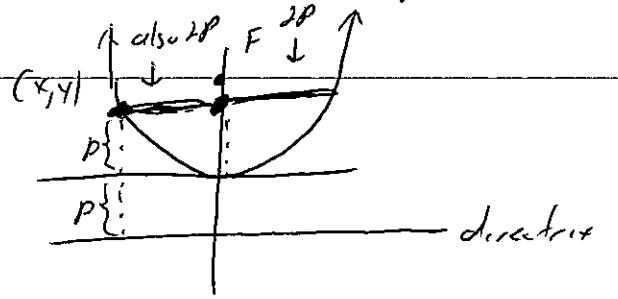
Graphing $y^2 = -6x$

$y = \pm\sqrt{-6x}$

$y = +\sqrt{-6x}$ and $y = -\sqrt{-6x}$

Find the focal diameter of a parabola

= width of parabola from focal point = $|4p|$



(x,y) equidistant from Focus and directrix

Definition — line segment that runs through the focus ⊥ to the axis with endpoints on the parabola.
= focal diameter.

Ex: Find Focus, Directrix, Focal Diameter of the parabola $y = \frac{1}{2}x^2$ and sketch

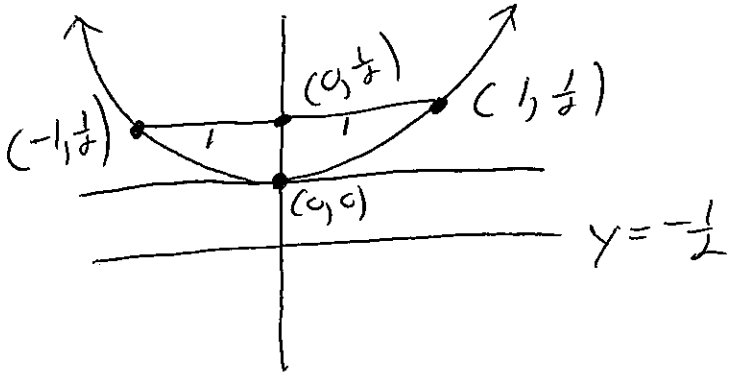
Get x^2 by itself for basic form of $x^2 = 4py$

$y = \frac{1}{2}x^2$

$\Rightarrow x^2 = 2y$ $\quad x^2 = 4py$ so $4p = 2 \Rightarrow p = \frac{1}{2}$

Focus = $(0, \frac{1}{2})$ Directrix = $y = -p = $ **$y = -\frac{1}{2}$**

Focal Diameter = $|4p| = |4(\frac{1}{2}| =$ **2 = Focal Diameter**



Shifting Parabolas

$(x-h)^2 = 4p(y-k)$ $p>0$ = points up, $p<0$ = points down

$(y-k)^2 = 4p(x-h)$ $p>0$ = opens right, $p<0$ = opens left

} Standard format

Ex: $x^2 - 4x = 8y - 28$

Get in standard format
complete the square
Get 1x or 1y by itself in ( ) on other side

$(x-2)^2 = 8y - 28 + 4$

$(x-2)^2 = 8y - 24$

$(x-2)^2 = 8(1y-3)$

by x is left/right shift

by y is up/down shift

$(x-2)^2 = 8(y-3)$ = Parabola shifted right 2, up 3 (2,3)

$(x-2)^2 = 8(y-3)$

Ignoring shift = $x^2 = 8y$ $x^2 = 4py$ $4p = 8$ $p = 2$

Basic V (0,0)
F (0,2)
Directrix $y = -2$

Shift (2,3) →

$V = (2,3)$
$F = (2,5)$
Directrix $y = 1$