# Math 12 H Section 10.2 Ellipses

Ellipse = set of all points in a plane whose <u>sum</u> of distances from <u>2</u> fixed points is constant.

The 2 fixed points are the <u>foci</u> of the ellipse.

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## Equation of ellipse. Center = <u>origin</u> (let $a$ = bigger # in each case)

Major axis = x-axis

$(a > b > 0)$

Foci on x-axis

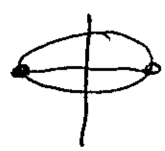
Equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(Bigger number <u>under x</u>)

Vertices = where ellipse crosses major axis (x)

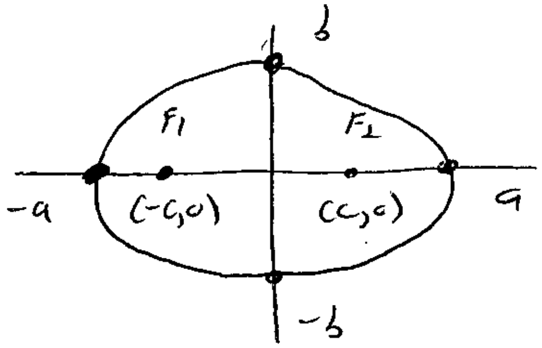
$(\pm a, 0)$



Major axis Horizontal length = $2a$

Minor axis Vertical length = $2b$

Foci $(\pm c, 0)$ $\underline{c^2 = a^2 - b^2}$




Major axis = y-axis

$a > b > 0$

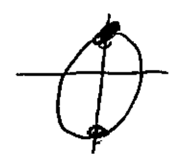
Foci on y-axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

(Bigger number <u>under y</u>)

Vertices = where ellipse crosses major axis (y)

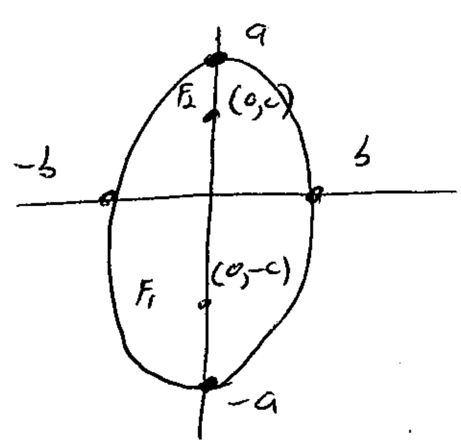
$(0, \pm a)$



Major axis Vertical length = $2a$

Minor axis Horizontal length = $2b$

Foci $(0, \pm c)$ $\underline{c^2 = a^2 - b^2}$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Ex: $\frac{x^2}{9}+\frac{y^2}{4}=1$

Find Foci, vertices, lengths major + minor axes, sketch, Graph on Calc

Denominator under $x^2$ is larger $9>4$, so major axis is horizontal.

$a^2=9 \quad a=3 \qquad b^2=4 \quad b=2$

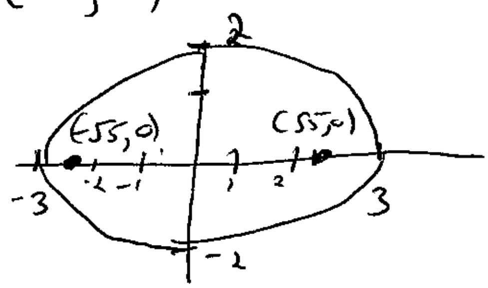
$c^2=a^2-b^2 \qquad c^2=9-4=5 \qquad c=\sqrt{5}$

$a=3$
$b=2$
$c=\sqrt{5}$

Foci $=(\pm\sqrt{5},0)$ vertices $(\pm 3,0)$

major axis $=2(a)=6$

minor axis $=2(b)=4$




Graph $\frac{x^2}{9}+\frac{y^2}{4}=1$ Get $y^2$ by itself.

$\frac{y^2}{4}=1-\frac{x^2}{9}$

$y^2=4\left(1-\frac{x^2}{9}\right)$

$y=\pm\sqrt{4\left(1-\frac{x^2}{9}\right)}$

$y=\pm 2\sqrt{1-\frac{x^2}{9}}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

**Ex:** $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Find Foci, vertices, lengths major + minor axes, sketch, Graph on Calc

Denominator under $x^2$ is larger $9 > 4$, so major axis is horizontal.

$a^2 = 9 \quad a = 3 \quad b^2 = 4 \quad b = 2$

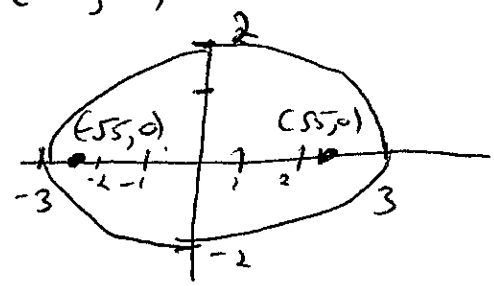
$c^2 = a^2 - b^2 \quad c^2 = 9 - 4 = 5 \quad c = \sqrt{5}$

$a = 3$
$b = 2$
$c = \sqrt{5}$

Foci $= (\pm\sqrt{5}, 0)$ vertices $(\pm 3, 0)$

major axis $= 2(a) = 6$

minor axis $= 2(b) = 4$




Graph $\frac{x^2}{9} + \frac{y^2}{4} = 1$ Get $y^2$ by itself.

$\frac{y^2}{4} = 1 - \frac{x^2}{9}$

$y^2 = 4\left(1 - \frac{x^2}{9}\right)$

$y = \pm\sqrt{4\left(1 - \frac{x^2}{9}\right)}$

$y = \pm 2\sqrt{1 - \frac{x^2}{9}}$

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Ellipse = set of all points in a plane whose sum of distances from 2 fixed points is constant.

The 2 fixed points are the foci of the ellipse.

p. 755 Equation of ellipse. Center = origin (let a = bigger # in each case)

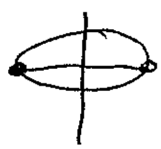
Major axis = x-axis
$(a > b > 0)$
Foci on x-axis

Equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ (Bigger number under x)

Vertices = where ellipse crosses major axis (x)

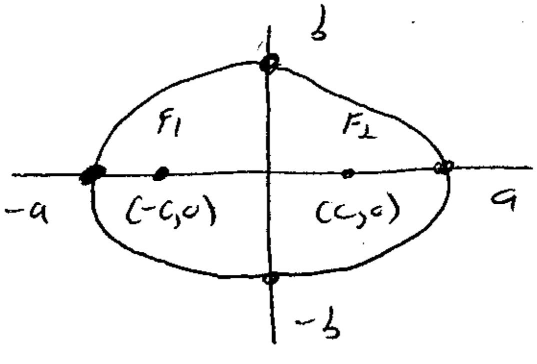
$(\pm a, 0)$



Major axis Horizontal length = $2a$

Minor axis Vertical length = $2b$

Foci $(\pm c, 0)$ $c^2 = a^2 - b^2$

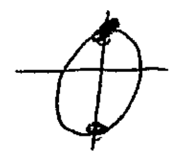



Major axis = y-axis
$a > b > 0$
Foci on y-axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$ (Bigger number under y)

Vertices = where ellipse crosses major axis (y)

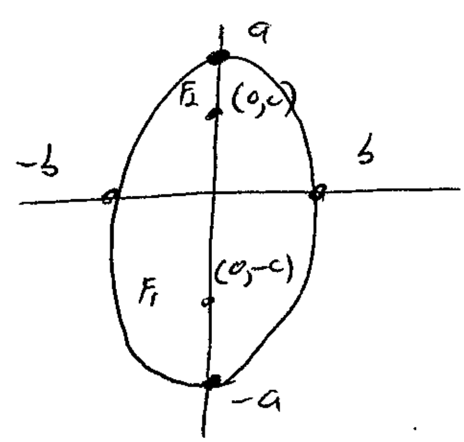
$(0, \pm a)$



Major axis Vertical length = $2a$

Minor axis Horizontal length = $2b$

Foci $(0, \pm c)$ $c^2 = a^2 - b^2$



Ex: Find the foci of the ellipse + sketch

$16x^2 + 9y^2 = 144$

Standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

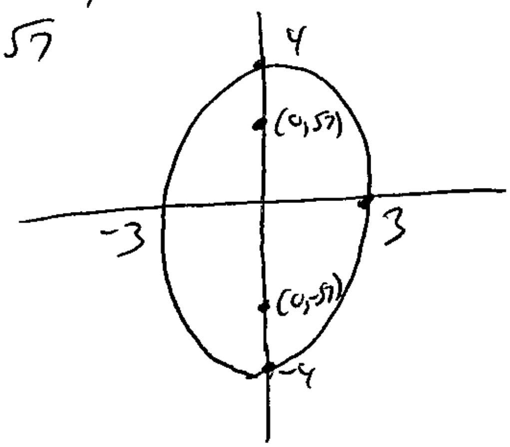
$\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144} = \frac{x^2}{9} + \frac{y^2}{16} = 1$

$a = 4$
$b = 3$

Bigger number under $y^2$, so foci is on y-axis

$c^2 = 16 - 9 \qquad c^2 = 7 \qquad c = \pm\sqrt{7}$

Focus = $(0, +\sqrt{7})$, $(0, -\sqrt{7})$



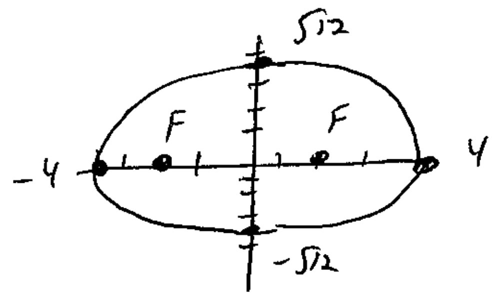

Ex)
Finding Equation of an ellipse $\frac{x^2}{a^2} = \frac{y^2}{b^2} = 1$

Vertices are $(\pm 4, 0)$ foci = $(\pm 2, 0)$ Find equation + sketch.

$a = 4$ from vertices $\qquad c = 2$ from foci $\qquad$ Find $b$

$c^2 = a^2 - b^2 \qquad 4 = 16 - b^2 \qquad b^2 = 16 - 4 \qquad b^2 = 12 \qquad b^2 = 12 \qquad b = \pm\sqrt{12} = \pm 2\sqrt{3}$

$\frac{x^2}{16} + \frac{y^2}{12} = 1$



Eccentricity of an ellipse — how "stretched" an ellipse is.

$e = \frac{c}{a}$ $(a > b > 0)$ and $c = \sqrt{a^2 - b^2}$ $= \frac{c}{\text{bigger number}}$

$0 < e < 1$ Bigger e = more stretch. smaller e = closer to a circle

Find an equation of an ellipse from its eccentricity and Foci

foci = $(0, \pm 8)$ $e = \left(\frac{4}{5}\right)$ Sketch

y = major axis

ratio
a not necessarily = 5

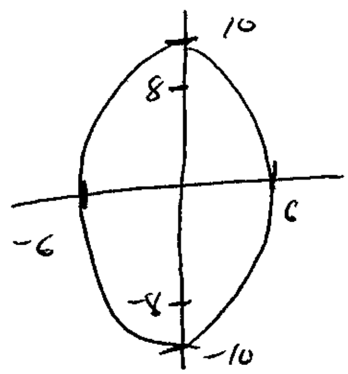
$\boxed{\begin{array}{c} c = 8 \\ \text{foci} \end{array}}$

$e = \frac{c}{a}$ $\frac{8}{a} = \frac{4}{5}$ $4a = 40$ $\underline{a = 10}$

$c^2 = a^2 - b^2$

$64 = 100 - b^2$
$b^2 = 100 - 64$
$b^2 = 36$, $b = 6$

$$\boxed{\frac{x^2}{36} + \frac{y^2}{100} = 1}$$




Shifted Ellipses $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Find original info then shift.

Find graph of $\frac{(x+1)^2}{4} + \frac{(y-2)^2}{9} = 1$

Find Foci of original

center = $(-1, 2)$
shift = $(-1, 2)$

original = $\frac{x^2}{4} + \frac{y^2}{9} = 1$

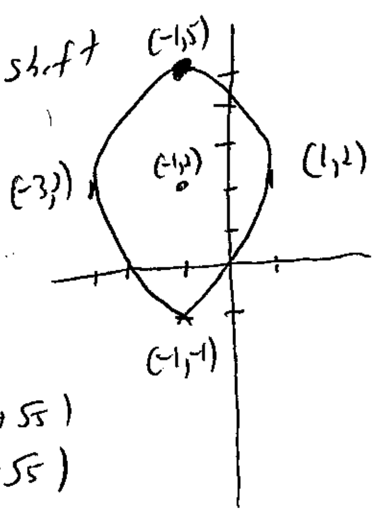
center = $(-1, 2)$ = shift

x end points =

$(2,0) \rightarrow (1, 2)$
$(-2,0) \rightarrow (-3, 2)$

y $(0,3) \rightarrow (-1, 5)$
y $(0,-3) \rightarrow (-1, -1)$

Foci $(0, \sqrt{5}) \rightarrow (-1, 2 + \sqrt{5})$
Foci $(0, -\sqrt{5}) \rightarrow (-1, 2 - \sqrt{5})$




now apply shift.

$C = (0, 0)$
$a = 3$, $b = 2$
$c^2 = 9 - 4 = 5$
$c = \pm\sqrt{5}$

Foci = $(0, \pm\sqrt{5})$

End points $(2, 0)$
$(-2, 0)$
$(0, 3)$
$(0, -3)$