



To sketch a hyperbola.

- 1) sketch the central box plot  $\pm a, \pm b$
- 2) sketch the asymptotes by diagonals going through central box.
- 3) plot vertices, (either  $2x$  or  $2y$  intercepts)  
must have both.
- 4) sketch the hyperbola.

Ex: A hyperbola has the equation  $9x^2 - 16y^2 = 144$

- a) Find the vertices, foci, asymptotes and sketch
- b) Graph with a calculator.

1) Get into standard form  $\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$

$a=4$   
 $b=3$   $\frac{x^2}{16} - \frac{y^2}{9} = 1$  = horizontal transverse ( $x^2 = +$ )  
vertices + foci on  $x$ -axis.

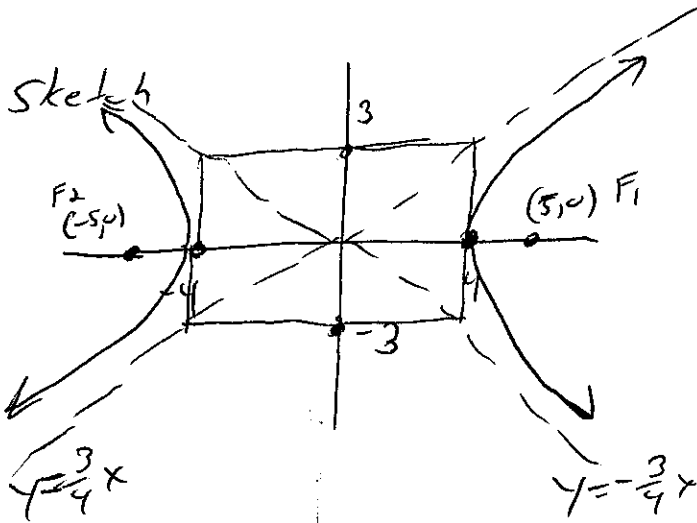
vertices =  $(\pm 4, 0)$

$a=4, b=3$

$c^2 = a^2 + b^2$   $c^2 = \sqrt{16+9}$   $c = \pm 5$

Foci =  $(\pm 5, 0)$

Asymptotes  $y = \pm \frac{b}{a}x = \pm \frac{3}{4}x$



Graph on Calc

= set  $y$  by itself

$9x^2 - 16y^2 = 144$

$\frac{-16y^2}{-16} = \frac{144 - 9x^2}{-16}$

$y^2 = -9 + \frac{9x^2}{16}$  factor out 9

$y^2 = 9(\frac{x^2}{16} - 1)$

$y = \pm \sqrt{9(\frac{x^2}{16} - 1)} = \pm 3 \sqrt{\frac{x^2}{16} - 1}$

Graph both

Ex: Find the equation and foci of the hyperbola with vertices  $(0, \pm 2)$  and asymptotes  $y = \pm 2x$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$a = 2$$

$$y = \pm \frac{a}{b}x \quad \frac{a}{b} = 2$$

$$\frac{2}{b} = 2$$

$$\frac{y^2}{4} - \frac{x^2}{1} = 1$$

$$\text{Focus } a^2 + b^2 = c^2$$

$$4 + 1 = c^2$$

$$c = \sqrt{5}$$

$$b = \underline{\underline{1}}$$

Foci =  $(0, \pm \sqrt{5})$ , Sketch.

### Shifting Hyperbolas

Find the center, foci, asymptotes of the hyperbola + sketch

$$9x^2 - 72x - 16y^2 - 32y = 16$$

(complete the square)

$$9(x^2 - 8x) - 16(y^2 + 2y) = 16$$

$$9(x-4)^2 - 16(y+1)^2 = 16 + \underline{9 \cdot 16} + \underline{-16(1)}$$

$$\frac{9(x-4)^2}{144} - \frac{16(y+1)^2}{144} = \frac{144}{144}$$

$$\frac{(x-4)^2}{16} - \frac{(y+1)^2}{9} = 1$$

Shift =  $(4, -1)$

Find all originals, then shift

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$v = (\pm 4, 0)$$

$$a = 4, b = 3$$

$$c^2 = a^2 + b^2$$

$$c = 5 \quad \text{Foci} = (\pm 5, 0)$$

$$\text{Asymptote } y = \pm \frac{b}{a}x$$

$$y = \pm \frac{3}{4}x$$

$$\text{Center} = (4, -1)$$

vertices =

$$(4, 0) + (4, -1) = (8, -1) \quad \text{vertices}$$

$$(-4, 0) + (4, -1) = (0, -1)$$

$$\text{Foci } (5, 0) + (4, -1) = (9, -1) \quad \text{Foci}$$

$$(-5, 0) + (4, -1) = (-1, -1)$$

Asymptotes  $y = \pm \frac{3}{4}x = \text{original}$

use point slope form to get

shifted  $C = (4, -1)$

$$(y-k) = \pm m(x-h)$$

$$(y+1) = \pm \frac{3}{4}(x-4) \leftarrow \text{solve}$$

# Hyperbolas

Ex: Find the vertices, foci, asymptotes and sketch

$$x^2 - 9y^2 + 9 = 0$$

Subtracting  $x^2$

$$\frac{x^2}{-9} - \frac{9y^2}{-9} = \frac{-9}{-9}$$

= vertical transverse axis

$$a = b, b = 3$$

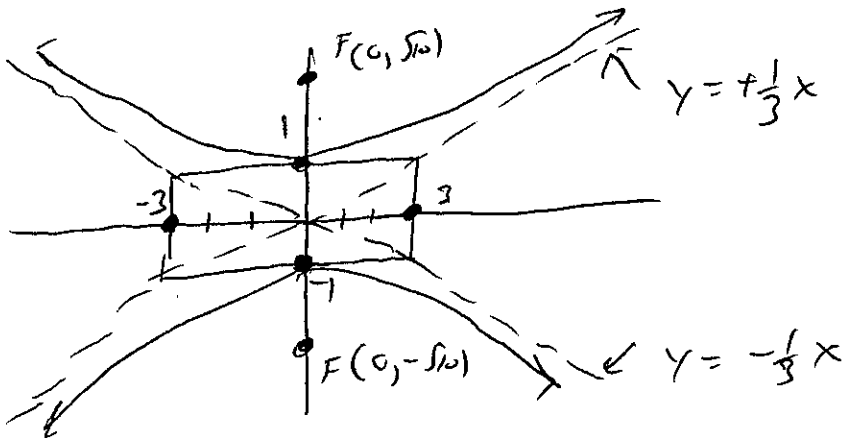
$$\frac{y^2}{1} - \frac{x^2}{9} = 1$$

$$\text{vertices} = (0, \pm 1)$$

$$\begin{aligned} \text{Foci } c^2 &= a^2 + b^2 \\ c^2 &= 1 + 9 \end{aligned}$$

$$c = \sqrt{10} \quad \text{Foci} = (0, \pm \sqrt{10})$$

$$\text{Asymptotes } y = \pm \frac{a}{b}x = \pm \frac{1}{3}x$$



$$\begin{aligned} \text{Calc: } \\ y &= \pm \sqrt{1 + \frac{x^2}{9}} \end{aligned}$$

Ex: Find Equation of a Hyperbola from vertices + foci, sketch  
 vertices =  $(\pm 3, 0)$     foci =  $(\pm 4, 0)$

$$a = 3, \quad b = ?$$

$$c = 4$$

$$a^2 + b^2 = c^2$$

$$9 + b^2 = 16$$

$$b = \sqrt{7}$$

$$\frac{x^2}{3^2} - \frac{y^2}{b^2} = 1$$

$$\boxed{\frac{x^2}{9} - \frac{y^2}{7} = 1}$$

