Math 12H Lesson Plan Parametric Equations. Section 10.7

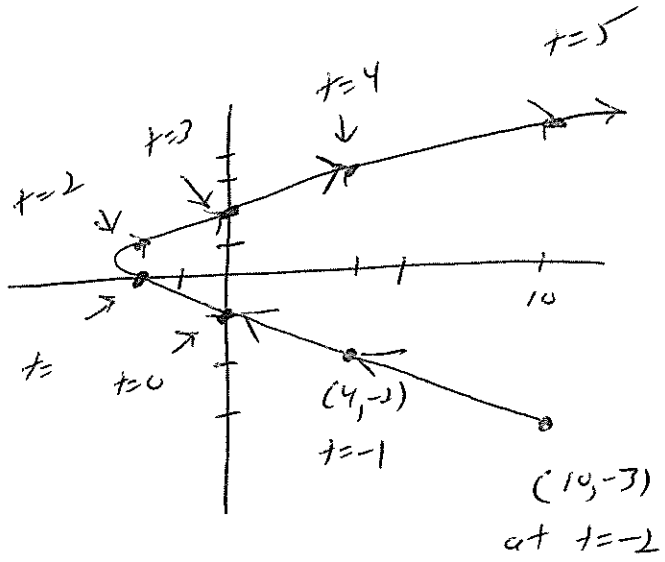
Sketching a curve defined by parametric equations.

Parametric equations are like rectangular equations with an additional time component. Where is the (x, y) coordinate at a given time t?

Ex: $x = t^2 - 3t$ $\quad y = t - 1$

E.g.

t	x	y
-2	10	-3
-1	4	-2
0	0	-1
1	-2	0
2	-2	1
3	0	2
4	4	3
5	10	4



Parametric shows which way an object is travelling (and how fast) along the path.

Replace t with $-t$ and the graph is the same but the arrows and graph trace out in the opposite direction.

Replace t with $2t$ → the graph is the same but it is traced out twice as fast.

Eliminating the parameter:

Writing a parametric equation as a single rectangular equation in terms of x and y.

Eliminate the parameter → get t by itself in one equation and then substitute it into the other.

Ex: $x = t^2 - 3t$ $\quad y = t - 1 \longrightarrow t = y + 1$

plug into x equation.

$x = t^2 - 3t$

$x = (y+1)^2 - 3(y+1)$

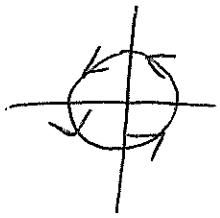
$x = y^2 + 2y + 1 - 3y - 3$

$x = y^2 - y - 2$

$\boxed{x = y^2 - y - 2}$: a parabola

Ex: $x = \cos t$ $\quad y = \sin t$ $\quad 0 \le t \le 2\pi$

$\cos^2 t + \sin^2 t = 1 \longrightarrow x^2 + y^2 = 1$



Ex: $x = \sin t$ $\quad y = 2 - \cos^2 t$ use identities to eliminate a parameter

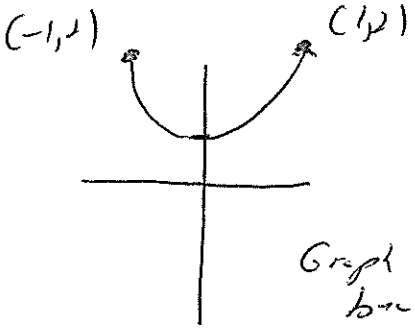
$\cos^2 t = 1 - \sin^2 t$ so $y = 2 - (1 - \sin^2 t) = 2 - 1 + \sin^2 t$

$y = 1 + \sin^2 t$ since $x = \sin t$

$y = 1 + x^2$ Since $-1 \le \sin t \le 1$

x is bounded

$-1 \le x \le 1$



Graph will oscillate back and forth.

Ex: Find the parametric equation for the line of slope 3 and passes through $(2, 6)$. (x y)

Slope $= \frac{3}{1}$ in terms of parametric $= \frac{3t}{1t}$

$$\boxed{x = 2 + t \qquad y = 6 + 3t}$$

In Rectangular $\qquad x = 2 + t \rightarrow \quad t = x - 2$

$$y = 6 + 3(x-2) = 6 + 3x - 6$$

$$\boxed{y = 3x}$$

Changing Polar Equations to Parametric form.

$x = r\cos\theta \qquad y = r\sin\theta$

$= f(\theta)\cos\theta \qquad = f(\theta)\sin\theta$

replace θ in polar with parametric variable of t

polar $\quad r = f(\theta)$ $\qquad$ parametric $\quad x = f(t)\cos t$

$y = f(t)\sin t$

$f(\theta)\cos\theta = t\cos t$

$\boxed{r = \theta}$ = polar

parametric $\rightarrow$ $\boxed{x = t\cos t \qquad y = t\sin t}$