Math 12 H Lesson Plan - Finding limits Algebraically - Section 12.2

Using calculators to guess the value of a limit doesn't always work. There are algebraic rules to calculate a limit. Given $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$

1) $$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$ Limit of a sum.

2) $$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$ Limit of a difference

3) $$\lim_{x \to a} [c \cdot f(x)] = c \cdot \lim_{x \to a} f(x)$$ Limit of a constant multiplier.

4) $$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$ Limit of a product.

5) $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$ if $\lim_{x \to a} g(x) \neq 0$. Limit of a quotient.

6) $$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$$ n is positive. Limit of a power

7) $$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$$ n is positive. Limit of a root.

Special Units

1) $\lim_{x \to a} c = c$ 2) $\lim_{x \to a} x = a$ 3) $\lim_{x \to a} x^n = a^n$

4) $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$.

Ex:) $\lim_{x \to 5} (2x^2 - 3x + 4) = 2(5)^2 - 3(5) + 4 = 39$

Ex:) $\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} = \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = \frac{-1}{11}$

General Rules for a limit as $x \to c$

1) Plug in value for x. If you do not get zero in denominator, the result is the limit.

2) If you plug in the value for x and you get 0 in the denominator, the graph is discontinuous at that value of x, but it might still have a limit.

Try to find the limit by factoring and cancelling, simplifying the expression, rationalizing, or checking left hand and right hand limits.

Ex: $\lim_{x\to 1} \frac{x-1}{x^2-1}$ Plug in $x=1$ = $\frac{0}{0}$ = undefined.

$\lim_{x\to 1} \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \frac{1}{x+1}$ = now plug in $x=1$ $= \frac{1}{1+1} = \frac{1}{2}$

Ex: $\lim_{h\to 0} \frac{(3+h)^2-9}{h}$ Plug in $h=0$ = $\frac{0}{0}$ = undefined.

$\lim_{h\to 0} \frac{(3+h)^2-9}{h} = \lim_{h\to 0} \frac{9+6h+h^2-9}{h} = \frac{6h+h^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} = 6+0 = \underline{\underline{6}}$

Ex: $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$

$= \lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} \cdot \frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3} = \lim_{t\to 0} \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)}$

$= \lim_{t\to 0} \frac{\cancel{t^2}}{\cancel{t^2}(\sqrt{t^2+9}+3)} = \frac{1}{\sqrt{t^2+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

Ex: $\lim_{x \to 0} |x|$ $\qquad |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$\lim_{x \to 0^-} |x| = 0$ and $\lim_{x \to 0^+} |x| = 0$

Since left hand limit $= 0 =$ right hand limit $= 0$

the $\lim_{x \to 0} |x| = 0$.

If can't reduce, rationalize, etc, check left-hand limit and right-hand limit and see if they are the same. If they are, you have the limit. If not, the limit does not exist.