Math 12 H Lesson Plan - Section 12.4 - Limits at infinity.

Limits at infinity - what happens to $f(x)$ as x gets arbitrarily large (either positive or negative).

Show example 2 p. 912.

Look to the highest power of x in $f(x)$. as $x \to \pm\infty$. One of 3 cases.

1) Biggest power of x is in the denominator.
   Limit $= 0$.

2) Biggest power of x is in the numerator.
   Limit = either + or $-\infty$ depending on signs.

3) Biggest powers equal in numerator and denominator
   Limit = ratio of lead coefficient of highest power
   will be positive or negative depending on signs of x and coefficients.

Ex: $\lim_{x \to \infty} \frac{1}{x}$ $\lim_{x \to -\infty} \frac{1}{x}$

Ex: $\lim_{x \to \infty} \frac{x^2-1}{x^2+1}$ $\lim_{x \to -\infty} \frac{x^2-1}{x^2+1}$

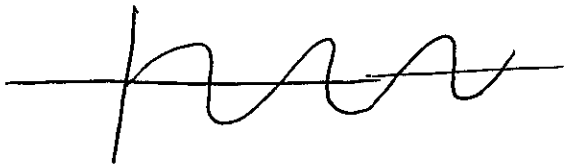
Ex $\lim_{x \to -\infty} \frac{-3x^3+17x^2+100}{-10x^3-5000x+1}$ $\lim_{x \to \infty} \frac{2x^2+7x-10}{8x^2-13x+7}$

$\lim_{x \to -\infty} \frac{5x^4+17x-1}{2x^4-1000x+6}$

Ex: $\lim_{x\to-\infty} \frac{4x^3-17x+7}{10x-8}$ $\lim_{x\to\infty} \frac{7x^6+3x-1}{-x^2+18x-2}$

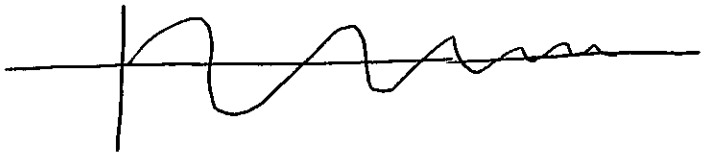
Some functions have no limit as $x \to \pm\infty$

Ex $\lim_{x\to\infty} \sin x$ = D.N.E.



Graph is not heading to any one value.

$\lim_{x\to\infty} \frac{\sin x}{x} = 0$



Ex: Limits of a Sequence

If a sequence's terms approaches a specific number, we say the sequence converges.

If a sequence's terms do not approach a specific number, we say a sequence is divergent.

Ex: $a_n = \frac{n}{n+1}$ $\lim_{n\to\infty} \frac{n}{n+1} = 1$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$ Sequence is convergent. Converges to 1.

Ex: $a_n = (-1)^n$ terms $= -1, 1, -1, 1, -1, 1, -1, 1, \ldots$

terms oscillate forever between -1 and 1

So sequence has no limit. It is divergent.

Math 12 H Lesson Plan - Section 12.4 Limits at infinity.

Ex: $a_n = \frac{15}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$

Lets find the biggest term in the numerator and denominator.

$$\frac{15(n)(n+1)(2n+1)}{n^3 \cdot 6} = \frac{15 \cdot n \cdot n \cdot 2n}{6n^3} = \frac{30n^3}{6n^3} = 5$$