Math 12H Lesson Plan Polar Coordinates - Section 8.1

System of coordinates we have used is the rectangular or Cartesian coordinates. It is described by an ordered pair (x, y), the distance of a point to two perpendicular axes.

Sometimes it's useful to represent a point from a specific fixed reference point.

The polar coordinate system describes a location in terms of distance from a single point or pole.

The location is given by an ordered pair (r, θ) where r is the distance from the origin or pole and θ is the angle from the positive x-axes.

You can convert back and forth between rectangular and polar form. It is very similar to converting between component and trig form of a vector.

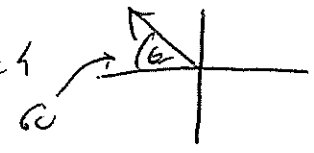
Ex rectangular coordinate = $(-2, 2\sqrt{3})$ x, y

change to polar = need (r, θ)

like changing $\langle -2, 2\sqrt{3} \rangle$ to trig form

$r^2 = x^2 + y^2$ $\quad r = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

$r = \sqrt{x^2+y^2}$

$\tan \theta = \frac{y}{x}$ $\quad$ sketch  $\quad \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 60° = \theta$

$\theta = 120° = \frac{2\pi}{3}$

$(-2, 2\sqrt{3}) = \left(4, \frac{2\pi}{3}\right)$ in polar form

Converting from polar to rectangular

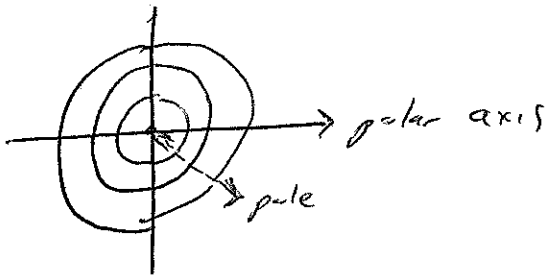
$x = r\cos\theta \qquad y = r\sin\theta \qquad = r(\cos\theta, \sin\theta)$

polar $= (4, \frac{2\pi}{3}) = 4(\cos\frac{2\pi}{3}, \sin\frac{2\pi}{3})$

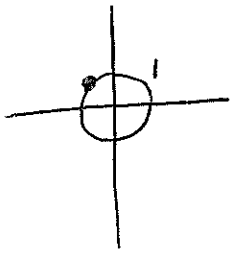
$= 4(-\frac{1}{2}, \frac{\sqrt{3}}{2}) = (-2, 2\sqrt{3})$

Distribution like vectors.

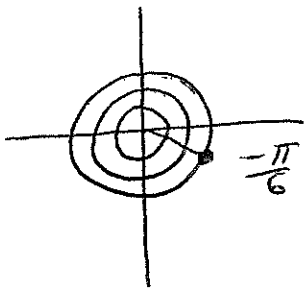
Plotting Points in Polar Coordinates



Ex: $(1, \frac{3\pi}{4})$



$(3, -\frac{\pi}{6})$

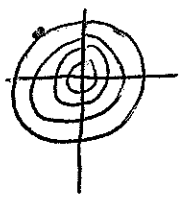


Distance is concentric circles from the center,
Point plotted on angle at appropriate circle.

Different polar coordinates can represent the same point. Ex: (r, θ) and $(-r, \theta+\pi)$ = same point.

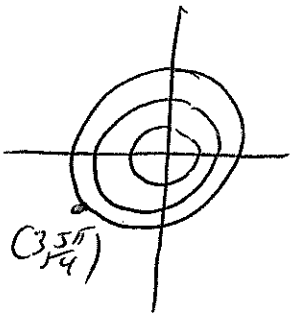
Ex: $(4, \frac{2\pi}{3}) = (-4, \frac{2\pi}{3}+\pi) = (-4, \frac{2\pi}{3}-\pi)$

$= (-4, \frac{5\pi}{3}) \quad = (-4, -\frac{\pi}{3})$



When plotting polar coordinates with a negative r-value, determine where the angle would be then reflect 180° to account for negative r-value

Ex: $(3, \frac{5\pi}{4})$ = $225°$



$(-3, \frac{5\pi}{4})$ = reflect $(3, \frac{5\pi}{4})$ in pole = move $180°$ or $\pm 1\pi$.