

## Section 8.1 Polar Equations

Converting Equations between Polar and rectangular systems.

Given  $(X, Y)$  in rectangular

$$X = r \cos \theta \text{ in polar} \quad Y = r \sin \theta \text{ in polar}$$

Going from a standard equation in rectangular form to polar

1) Change  $x$  to  $r \cos \theta$  and 2) substitute  $y = r \sin \theta$ .

3) Simplify as much as possible; Either solving for  $\theta$  or for  $r$ .

Ex:  $X^2 = 4y$   $x = r \cos \theta$  depending on what can be eliminated,  $y = r \sin \theta$

$$\rightarrow (r \cos \theta)^2 = 4(r \sin \theta)$$

$$\rightarrow \frac{r^2 \cos^2 \theta = 4r \sin \theta}{r \cos^2 \theta} \quad \text{Get } r \text{ by itself}$$

$$\rightarrow r = \frac{4 \sin \theta}{\cos^2 \theta} = 4 \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = 4 \tan \theta \sec \theta$$

$$\boxed{r = 4 \tan \theta \sec \theta}$$

Ex:  $X = Y$

$$\frac{r \cos \theta}{r \cos \theta} = \frac{r \sin \theta}{r \cos \theta}$$
$$1 = \tan \theta$$

$$\boxed{\theta = \frac{\pi}{4}}$$

Converting from polar to rectangular form of an equation.

Using substitutions to get rid of  $r$  and  $\theta$ .

Make terms into  $r^2$  or  $r \cos \theta$ ,  $r \sin \theta$ .

Want to get  $x$  and  $y$

$$x = \boxed{r \cos \theta}, \quad y = \boxed{r \sin \theta}$$

then substitute

Ex:  $r = 5 \sec \theta$       $r = \frac{5}{\cos \theta}$       $\underline{\underline{r \cos \theta = 5}}$

$$\boxed{x = 5}$$

Ex:  $r = 2 \sin \theta$      multiply both sides by  $r$

$$\underline{\underline{r^2 = 2 r \sin \theta}}$$

$$\frac{x^2 + y^2 = 2y}{\downarrow \text{keep } x \text{ only}}$$

$$x^2 + y^2 - 2y = 0$$

$$\underline{\underline{x^2 + (y-1)^2 = 1}}$$

Ex:

$$r = 2 + 2 \cos \theta \quad \text{multiply by } r$$

$$\underline{\underline{r^2 = 2r + 2r \cos \theta}}$$

$$x^2 + y^2 = 2x + 2(x)$$

$$x^2 + y^2 - 2x = 2x$$

$$(x^2 + y^2 - 2x)^2 = 4x^2$$

$$\underline{\underline{(x^2 + y^2 - 2x)^2 = 4(x^2 + y^2)}}$$