Math 12H Lesson Plan - Matrices - Section 9.4 Page 1

A system of linear equations can be solved by using Matrices. <u>Matrix</u> = a rectangular array of numbers.

A matrix has a size called an order or dimension. The size of a matrix is expressed in terms of <u>rows</u> and <u>columns</u>. An $m \times n$ matrix has m rows and n columns.

Ex: A 3×4 matrix has 3 Rows and 4 columns.

$$\begin{array}{c|c} & \begin{matrix} C_1 & C_2 & C_3 & C_4 \end{matrix} \\ \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} & \begin{bmatrix} 4 & 2 & -1 & 8 \\ 5 & 3 & 1 & 7 \\ 10 & 2 & 0 & -6 \end{bmatrix} \end{array}$$

Ex: Matrix $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{bmatrix} = 2 \times 3$ Dimension

A specific element in matrix A is represented by a_{ij} where i = row number and j = column number.

Ex: $a_{12} = 3$

A System of equations can be represented using matrices.

Ex: $$\begin{aligned} 3x - 2y + z &= 5 \\ x + 3y - z &= 0 \\ -x \quad + 4z &= 11 \end{aligned}$$ in Matrix form =

Coefficient Matrix = $\begin{bmatrix} 3 & -2 & 1 \\ 1 & 3 & -1 \\ -1 & 0 & 4 \end{bmatrix}$

Augmented Matrix = $\begin{bmatrix} 3 & -2 & 1 & 5 \\ 1 & 3 & -1 & 0 \\ -1 & 0 & 4 & 11 \end{bmatrix}$

Columns represent coefficients of x, y and z.

includes variables coefficients and constants

Can solve a system of equations using matrices with the same 3 basic row operations.

1) Add a multiple of one row to another
2) Multiply a row by a constant
3) Interchange two rows.

<u>Row - Echelon form</u>

1) The first non-zero number in each row is 1.
2) The leading entry in each row is to the right of the row immediately above it.
3) All rows consisting entirely of zeros are at the bottom.

+4) Reduced-row echelon → every number <u>above</u> and <u>below</u> each leading entry of 1 is a zero.

Solving a system of equations using matrices and <u>Gauss</u> – <u>Jordan</u> Elimination = reduced row echelon form.

Ex:
$$\begin{aligned} 4x + 8y - 4z &= 4 \\ 3x + 8y + 5z &= -11 \\ -2x + y + 12z &= -17 \end{aligned}$$

$$\left[\begin{array}{cccc} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{array}\right] \begin{array}{l} R_1 \div 4 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{array}\right] \begin{array}{l} \\ -3R_1 + R_2 \rightarrow \\ 2R_1 + R_3 \end{array} \left[\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{array}\right] \begin{array}{l} \\ R_2 \div 2 \\ R_3 \div 5 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 1 & 2 & -3 \end{array}\right] \begin{array}{l} \\ \\ -R_2 + R_3 \end{array} \quad \left[\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -2 & 4 \end{array}\right] \begin{array}{l} \\ \\ R_3 \div -2 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{-2R_2 + R_1} \left[\begin{array}{cccc} 1 & 0 & -9 & 15 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} 9R_3 + R_1 \\ -4R_3 + R_2 \\ \\ \end{array}$$

Row echelon form

Can back substitute to solve for x, y, z

$$\left[\begin{array}{cccc} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \qquad \begin{array}{l} x = -3 \\ y = 1 \\ z = -2 \end{array}$$

Reduced Row Echelon

Just like solving a system of equations, solving using matrices can have

a) One solution

b) No solution = false statement = inconsistent

c) Infinite solutions = true statement = dependent equations.