Math 12 A Lesson Plan - Section 9.5 Algebra of Matrices

Two Matrices are equal if they have the same entries in the same positions. Obviously must also be the same size.

$A = B$ $\qquad\qquad$ $A \neq B$

$$\begin{bmatrix} 2 & 0 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{10}{5} & 0 \\ -\sqrt{9} & \sqrt{16} \end{bmatrix} \qquad \begin{bmatrix} 4 & 7 \\ 2 & 0 \\ -1 & 5 \end{bmatrix} \neq \begin{bmatrix} 4 & 7 \\ 2 & \sqrt{0} \\ -1 & 6 \end{bmatrix}$$

Sum / Difference / Scalar product of Matrices

When adding or subtracting 2 Matrices

1) Matrices <u>must</u> be of the same size
2) add or subtract corresponding elements in the 2 matrices.

Scalar Multiplication = multiply every element in the matrix by a constant.

Ex. $A = \begin{bmatrix} 4 & 6 \\ -2 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & 7 \\ -3 & 1 \\ 0 & 5 \end{bmatrix}$ $\quad C = \begin{bmatrix} -1 & -8 \\ 7 & 4 \\ 6 & 3 \end{bmatrix}$

$A+B =$ D.N.E $\quad B+C = \begin{bmatrix} 1 & -1 \\ 4 & 5 \\ 6 & 8 \end{bmatrix}$ $\quad B-C = \begin{bmatrix} 3 & 15 \\ -10 & -3 \\ -6 & 2 \end{bmatrix}$

$= \phi$

$$2A = 2\begin{bmatrix} 4 & 6 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ -4 & 6 \end{bmatrix}$$

Properties of Addition ... Scalar Multiplication

$A + B = B + A$ commutative

$(A+B) + C = A + (B+C)$ Associative

Given constants c and d

$c \cdot (dA) = (cd)(A)$ Associative prop of Scalar Multiplication

$(c+d)A = cA + dA$ Distributive Property of Scalars

$c(A+B) = cA + cB$

Ex: Solving a Matrix Equation

Solve for <u>Matrix</u> X

Given $A = \begin{bmatrix} 2 & 3 \\ -5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}$

$2X - A = B \longrightarrow 2X = B + A$ $X = \dfrac{A+B}{2}$

then $A+B = \begin{bmatrix} 6 & 2 \\ -4 & 4 \end{bmatrix}$ $\dfrac{A+B}{2} = \begin{bmatrix} \frac{6}{2} & \frac{2}{2} \\ -\frac{4}{2} & \frac{4}{2} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix} = X$

Matrix Multiplication

When multiplying a matrix A times Matrix B

The elements in the rows of matrix A are multiplied by the elements in the columns of Matrix B.

Given

$$\underbrace{\overset{A}{m \times \underbrace{n \quad \cdot \quad \overset{B}{n}}_{\text{must be same}} \times p}}_{\text{Size of new matrix}}$$

Ex: $A = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix}$

$C = \begin{bmatrix} 4 & -2 \\ 3 & 5 \\ 1 & 6 \end{bmatrix}$

Ex $A \times C$

$2\times2 \cdot 3\times2$ = can't be done = ϕ = D.N.E.

✓ $C \cdot A$

$$\underbrace{3 \times \underbrace{2 \cdot 2}_{\checkmark} \times 2}_{\text{new size}}$$

$$\begin{bmatrix} 4 & -2 \\ 3 & 5 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 4+2 & 12+0 \\ 3+-5 & 9+0 \\ 1+-6 & 3+0 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ -2 & 9 \\ -5 & 3 \end{bmatrix}$$

Positions in new matrix tell what column and Row were used to get that value.

$C \cdot B$

$$\begin{bmatrix} 4 & -2 \\ 3 & 5 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} \overset{-4+0}{\underline{-4}} & \overset{20-8}{\underline{12}} & \overset{8-14}{\underline{-6}} \\ \overset{-3+0}{\underline{-3}} & \overset{15+20}{\underline{35}} & \overset{6+35}{\underline{41}} \\ \overset{-1+0}{\underline{-1}} & \overset{5+24}{\underline{29}} & \overset{2+42}{\underline{44}} \end{bmatrix}$$

$3 \times 2 \cdot 2 \times 3$

new size

$B \cdot C$

$$\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ 3 & 5 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} \overset{-4+15+2}{\underline{13}} & \overset{2+25+12}{\underline{39}} \\ \underset{0+12+7}{\underline{19}} & \underset{0+20+42}{\underline{62}} \end{bmatrix}$$

$2 \times 3 \cdot 3 \times 2$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -1 & -3 \end{bmatrix}$$