

# Math 12 H Lesson Plan - Section 9.6 - Inverses

Identity Matrix - a square matrix with 1's along the diagonal and zero everywhere else.

Ex:  $2 \times 2$  Identity Matrix  $3 \times 3$  Identity Matrix etc.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$4 \times 4$

$I$  = identity Matrix.

Any Matrix  $A$  times the identity  $= A$

$$A \cdot I = A \quad I \cdot A = A$$

Ex  $A = \begin{bmatrix} 4 & 7 \\ -2 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -2 & 5 \end{bmatrix} \quad A \cdot I = A$

The Inverse of a matrix  $A = A^{-1}$  = "A inverse"  
A matrix times its inverse equals the identity matrix and vice-versa.

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

Ex: Show that  $B$  is the inverse of  $A$ .

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B \cdot A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot B = I \quad \text{and} \quad B \cdot A = I \quad \text{so } B \text{ is the inverse of } A.$$

Finding the inverse of a  $2 \times 2$  matrix

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$ad-bc = |A| = \text{Determinant of matrix } A.$

Ex: Find Inverse of  $\begin{bmatrix} 4 & 5 \\ -2 & 3 \end{bmatrix}$

$$A^{-1} = \frac{1}{(4)(3) - (5)(-2)} \cdot \begin{bmatrix} 3 & -5 \\ 2 & 4 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 3 & -5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{22} & -\frac{5}{22} \\ \frac{1}{11} & \frac{2}{11} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{22} & -\frac{5}{22} \\ \frac{1}{11} & \frac{2}{11} \end{bmatrix}$$

Check  $A \cdot A^{-1} = I$   
and  $A^{-1} \cdot A = I$  ✓

Finding the Inverse of a  $3 \times 3$  Matrix

$$A = \begin{bmatrix} 1 & -2 & -4 & | & 1 & 0 & 0 \\ 2 & -3 & -6 & | & 0 & 1 & 0 \\ -3 & 6 & 15 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{original}} \begin{array}{l} -2R_1 + R_2 \\ 3R_1 + R_3 \end{array} \begin{bmatrix} 1 & -2 & -4 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 3 & | & 3 & 6 & 1 \end{bmatrix} \xrightarrow{\text{Identity}} \begin{array}{l} 2R_2 + R_1 \\ 2R_3 + R_2 \end{array} \begin{bmatrix} 1 & 0 & 0 & | & -3 & 2 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 3 & | & 3 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \div 3} \begin{bmatrix} 1 & 0 & 0 & | & -3 & 2 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 0 & \frac{1}{3} \end{bmatrix} \xrightarrow{-2R_3 + R_2} \begin{bmatrix} 1 & 0 & 0 & | & -3 & 2 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 0 & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -3 & 2 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 0 & \frac{1}{3} \end{bmatrix} \xrightarrow{R_3 \div 3} \begin{bmatrix} 1 & 0 & 0 & | & -3 & 2 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 0 & \frac{1}{3} \end{bmatrix} \xrightarrow{-2R_3 + R_2} \begin{bmatrix} 1 & 0 & 0 & | & -3 & 2 & 0 \\ 0 & 1 & 2 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 1 & 0 & \frac{1}{3} \end{bmatrix}$$

Checking  $A \cdot A^{-1} = A^{-1} \cdot A = I$

Put the identity matrix at end of original. Get the original into Gauss-Jordan form while simultaneously doing the same changes to the identity matrix section. When complete, the right side will be the inverse of the original matrix.