Math 12 Lesson Plan - Section 9.7 - Determinants.

If a matrix is a square, it has a number associated with it called a determinant. Determinants can be used to solve equations and determine if there is an inverse.

To find the determinant of a 2x2 matrix

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ determinant of A $= \det(A) = |A|$

$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Find $|A|$ if $A = \begin{bmatrix} 6 & -3 \\ 2 & 3 \end{bmatrix}$ $= 6(3) - (-3)(2)$
$= 18 - -6 = 24$

Find the determinant of an $n \times n$ matrix using minors and cofactors.

The minor M_{ij} of an element a_{ij} is the determinant of the matrix obtained by deleting the i row and j column.
The cofactor $A_{ij} = (-1)^{i+j} \cdot M_{ij}$

Ex: $A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}$

$M_{12} = \begin{vmatrix} 0 & 4 \\ -2 & 6 \end{vmatrix} = 0 - (-8) = \underline{8} = \underline{\text{Minor of } M_{12}}$

Cofactor $A_{12} = (-1)^{1+2} \cdot M_{12}$
$= -1 \cdot (8) = \underline{-8}$

Ex: $\begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}$

$M_{33} = \begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = 4 - 0 = 4$

$A_{33} = (-1)^{3+3} \cdot M_{33} = 1 \cdot 4 = 4$

Going from Minor to Co Factor either multiply by 1 or -1.

Can Find the Determinant of any $n \times n$ matrix by using minors and cofactors. (one row or column)

Finding the Determinant of a 3×3 matrix

$\begin{bmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{bmatrix}$

$\begin{vmatrix} 2 & 3 & -1 & 2 & 3 \\ 0 & 2 & 4 & 0 & 2 \\ -2 & 5 & 6 & -2 & 5 \end{vmatrix}$

3 diagonal Left to Right − 3 diagonal Right to Left

$[(2)(2)(6) + (3)(4)(-2) + (-1)(0)(5)] - [(3)(0)(6) + (2)(4)(5) + (-1)(2)(-2)]$

$[24 + -24 + 0] - [0 + 40 + 4] = 0 - 44 = \boxed{-44}$