Math 12 H - Lesson Plan - Matrix Applications Section 9.7

Using Determinants to solve equations - Cramer's Rule.

Given a system of n linear equations with n variables,

$$x = \frac{|D_x|}{|D|} \qquad y = \frac{|D_y|}{|D|} \qquad z = \frac{|D_z|}{|D|} \qquad \text{etc.}$$

where $|D|$ is the determinant of the coefficient matrix and $|D_x|$, $|D_y|$, $|D_z|$, etc is the determinant of the coefficient matrix with the subscripted column replaced with the constants column.

Ex: $2x + 6y = -1$
$x + 8y = 2$

$$|D| = \begin{vmatrix} 2 & 6 \\ 1 & 8 \end{vmatrix} = 16 - 6 = 10$$

$$|D_x| = \begin{vmatrix} -1 & 6 \\ 2 & 8 \end{vmatrix} = -8 - 12 = -20 \qquad |D_y| = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4 - -1 = 5$$

$$x = \frac{|D_x|}{|D|} = \frac{-20}{10} = -2 \qquad y = \frac{|D_y|}{|D|} = \frac{5}{10} = \frac{1}{2}$$

$x = -2$, $y = \frac{1}{2}$ ✓

Works for any $n \times n$ matrix:

Ex:
$$\begin{aligned} 2x - 3y + 4z &= 1 \\ x \quad\quad + 6z &= 0 \\ 3x - 2y \quad\quad &= 5 \end{aligned}$$

$$|D| = \begin{vmatrix} 2 & -3 & 4 & 2 & -3 \\ 1 & 0 & 6 & 1 & 0 \\ 3 & -2 & 0 & 3 & -2 \end{vmatrix}$$

$$(0 + -54 + -8) - (0 + -24 + 0)$$
$$-62 - (-24) = \underline{\underline{-38}}$$

$$|D_x| = \begin{vmatrix} 1 & -3 & 4 & 1 & -3 \\ 0 & 0 & 6 & 0 & 0 \\ 5 & -2 & 0 & 5 & -2 \end{vmatrix}$$

$$(0 + -90 + 0) - (0 + -12 + 0)$$
$$-90 - (-12) = \underline{\underline{-78}}$$

$$|D_y| = \begin{vmatrix} 2 & 1 & 4 & 2 & 1 \\ 1 & 0 & 6 & 1 & 0 \\ 3 & 5 & 0 & 3 & 5 \end{vmatrix}$$

$$(0 + 18 + 20) - (0 + 60 + 0)$$
$$38 - 60 = -22$$

$$|D_z| = \begin{vmatrix} 2 & -3 & 1 & 2 & -3 \\ 1 & 0 & 0 & 1 & 0 \\ 3 & -2 & 5 & 3 & -2 \end{vmatrix}$$

$$(0 + 0 + -2) - (-15 + 0 + 0)$$
$$-2 - (-15) = +13$$

$$x = \frac{|D_x|}{|D|} = \frac{-78}{-38} = \frac{39}{19}$$

$$y = \frac{|D_y|}{|D|} = \frac{-22}{-38} = \frac{11}{19}$$

$$z = \frac{|D_z|}{|D|} = \frac{-13}{-38} = -\frac{13}{38}$$

Finding the area of a triangle using determinants.

Ex: Given Points $(a_1, b_1)$, $(a_2, b_2)$, $(a_3, b_3)$

$$\text{area} = \pm \frac{1}{2} \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

Given Points $(-1, 4)$ $(3, 6)$ $(1, 2)$ Find the area.

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} -1 & 4 & 1 \\ 3 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} \begin{vmatrix} -1 & 4 & 1 & -1 & 4 \\ 3 & 6 & 1 & 3 & 6 \\ 1 & 2 & 1 & 1 & 2 \end{vmatrix}$$

$$= \pm \frac{1}{2} (-6 + 4 + 6) - (12 + -2 + 6)$$

$$= \pm \frac{1}{2} (4) - (16) = \pm \frac{1}{2}(-12) = \boxed{6}$$

use $\left(\frac{-1}{2}\right)(-12)$ to get $+6$